Chapter 3 roadmap:

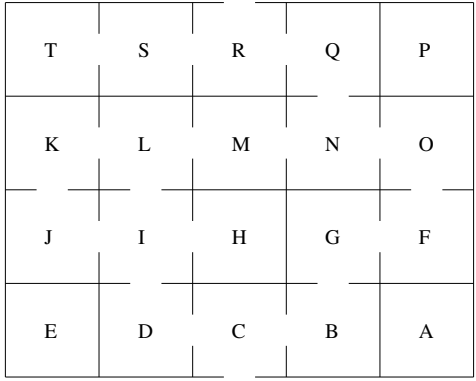
- Propositions, booleans, logical equivalence. §3.1 (last week Friday)
- Boolean sequences §3.2 (Monday)
- Conditional propositions and arguments. §3.3 (**Today**)
- Predicates and quantification. §3.(4 & 5) (Friday)
- (Begin proofs next week)

Today:

- Define conditional propositions
- Define arguments
- Consider known argument forms
- Practice verifying argument forms (Game 2)

p	q	$p \wedge q$	$p \vee q$	$\sim p$	$\sim p \vee q == p \rightarrow q$
T	T	T	T	F	T
T	F	F	T	F	F
F	T	F	T	T	T
F	F	F	F	T	T

p	q	(original) conditional $p \rightarrow q$	converse $q \rightarrow p$	inverse $\sim p \rightarrow \sim q$	contrapositive $\sim q \rightarrow \sim p$	negation $p \wedge \sim q$	biconditional $p \leftrightarrow q$
T	T	T	T	T	T	F	T
T	F	F	T	T	F	T	F
F	T	T	F	F	T	F	F
F	F	T	T	T	T	F	T



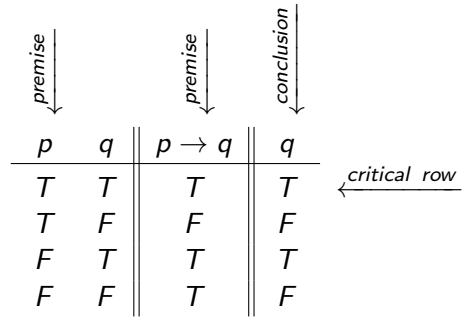
1. Bob passed through P.
2. Bob passed through N.
3. Bob passed through M.
4. If Bob passed through O, then Bob passed through F.
5. If Bob passed through K, then Bob passed through L.
6. If Bob passed through L, then Bob passed through K.

Based on example by Susanna Epp, 2006

Valid argument

If it is Monday, then it is raining
It is Monday.
Therefore it is raining.

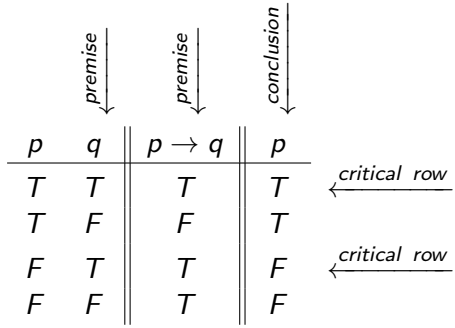
$p \rightarrow q$
p
$\therefore q$



Invalid argument

If it is raining, then there are clouds
There are clouds.
Therefore it is raining.

$p \rightarrow q$
q
$\therefore p$



Modus tollens

If it is spring, then the daffodils bloom.
The daffodils aren't blooming.
Therefore it is not spring.

p	q	$p \to q$	$\sim q$	$\sim p$
T	T	T	F	
T	F	F	T	
F	T	T	F	
F	F	T	T	T

Modus ponens
$p \to q$
p
$\therefore q$

Modus tollens
$p \to q$
$\sim q$
$\therefore \sim p$

Contradiction
$p \to F$
$\therefore \sim p$

Generalization
p
$\therefore p \vee q$

Specialization
$p \wedge q$
$\therefore p$

Elimination
$p \vee q$
$\sim p$
$\therefore q$

Conjunction
p
q
$\therefore p \wedge q$

Transitivity
$p \to q$
$q \to r$
$\therefore p \to r$

Division into cases
$p \vee q$
$p \to r$
$q \to r$
$\therefore r$

Arguments in literature

Elmination:

> *If anyone knows anything about anything, it's Owl who knows something about something, or my name isn't Winnie-the-Pooh. Which it is. So there you have it.* *A. A. Milne, Winnie-the-Pooh, Ch 4.*

Division into cases:

> *Soon her eye fell on a little glass box that was lying under the table: she opened it, and found in it a very small cake, on which the words "EAT ME" were beautifully marked in currants. "Well, I'll eat it," said Alice, "and if it makes me grow larger, I can reach the key; and if it makes me grow smaller, I can creep under the door; so either way I'll get into the garden, and I don't care which happens!"* *Lewis Carroll, Alice's Adventures in Wonderland, Ch 1.*

Proof by contradiction

$p \to F$
$\therefore \sim p$

p	$p \to F$	$\sim p$	
T	F	F	
F	T	T	$\xleftarrow{\text{critical row}}$

Restore us to yourself, O LORD, *that we may be restored. Renew our days as of old—unless you have utterly rejected us, and you remain exceedingly angry with us.* *Lam 5:21–22 (ESV)*

Mod Pon	Mod Tol	Generalization	Specialization	Elimination	Transitivity	Div into cases	Contradiction
$p \to q$	$p \to q$	p	$p \wedge q$	$p \vee q$	$p \to q$	$p \vee q$	$p \to F$
p	$\sim q$	$\therefore p \vee q$	$\therefore p$	$\sim p$	$q \to r$	$p \to r$	$\therefore \sim p$
$\therefore q$	$\therefore \sim p$			$\therefore q$	$\therefore p \to r$	$q \to r$	
						$\therefore r$	

3.3.17

(a) $t \to u$

(b) $p \vee \sim q$

(c) $p \to (u \to r)$

(d) q

(e) $\therefore t \to r$

Mod Pon	Mod Tol	Generalization	Specialization	Elimination	Transitivity	Div into cases	Contradiction
$p \to q$	$p \to q$	p	$p \wedge q$	$p \vee q$	$p \to q$	$p \vee q$	$p \to F$
p	$\sim q$	$\therefore p \vee q$	$\therefore p$	$\sim p$	$q \to r$	$p \to r$	$\therefore \sim p$
$\therefore q$	$\therefore \sim p$			$\therefore q$	$\therefore p \to r$	$q \to r$	
						$\therefore r$	

3.3.18

(a) $p \to t$

(b) $\sim (q \to t) \to w$

(c) $p \vee q$

(d) $\sim w$

(e) $\therefore t$

Mod Pon	Mod Tol	Generalization	Specialization	Elimination	Transitivity	Div into cases	Contradiction
$p \to q$	$p \to q$	p	$p \wedge q$	$p \vee q$	$p \to q$	$p \vee q$	$p \to F$
p	$\sim q$	$\therefore p \vee q$	$\therefore p$	$\sim p$	$q \to r$	$p \to r$	$\therefore \sim p$
$\therefore q$	$\therefore \sim p$			$\therefore q$	$\therefore p \to r$	$q \to r$	
						$\therefore r$	

3.3.24

(a) w

(b) $q \to r$

(c) $t \to s$

(d) $u \to s$

(e) $(\sim t \wedge \sim u) \to \sim w$

(f) $(s \vee y) \to (p \to q)$

(g) $\sim (p \to r) \vee x$

(h) $\therefore x$

Mod Pon	Mod Tol	Generalization	Specialization	Elimination	Transitivity	Div into cases	Contradiction
$p \to q$	$p \to q$	p	$p \wedge q$	$p \vee q$	$p \to q$	$p \vee q$	$p \to F$
p	$\sim q$	$\therefore p \vee q$	$\therefore p$	$\sim p$	$q \to r$	$p \to r$	$\therefore \sim p$
$\therefore q$	$\therefore \sim p$			$\therefore q$	$\therefore p \to r$	$q \to r$	
						$\therefore r$	

3.3.27

(a) $p \to q$

(b) x

(c) $\sim (p \vee w) \to r$

(d) $q \to u$

(e) $x \to t$

(f) $w \to u$

(g) $r \vee s$

(h) $r \to F$

(i) $\therefore t \wedge s \wedge u$

Mod Pon	Mod Tol	Generalization	Specialization	Elimination	Transitivity	Div into cases	Contradiction
$p \to q$	$p \to q$	p	$p \wedge q$	$p \vee q$	$p \to q$	$p \vee q$	$p \to F$
p	$\sim q$	$\therefore p \vee q$	$\therefore p$	$\sim p$	$q \to r$	$p \to r$	$\therefore \sim p$
$\therefore q$	$\therefore \sim p$			$\therefore q$	$\therefore p \to r$	$q \to r$	
						$\therefore r$	

3.3.25

(a) $u \to \sim p$

(b) $(\sim p \vee q) \to (r \to s)$

(c) $u \wedge \sim w$

(d) $t \to s$

(e) $(\sim t \wedge \sim r) \to w$

(f) $\therefore s$

For next time:

Do Exercises 3.5.(7-10) and 3.5.(3, 9-13)

Read Section 3.(6 & 7).

Take quiz