Chapter 6 outline:

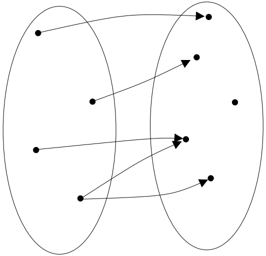
- Introduction, function equality, and dictionaries (last week Wednesday)
- Image and inverse images (last week Friday)
- Function properties and composition (Monday)
- Reducing and pipelining (Wednesday)
- Cardinality (**Today**)
- Countability and practice quiz (next week Monday)
- Review (next week Wednesday)
- Test 3, on Ch 5 & 6 (next week Friay, Nov 21)

Today:

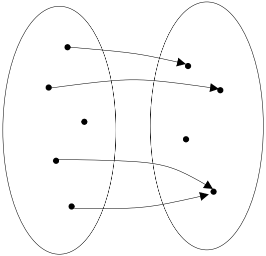
- Hints on previous HW problems
- Formal definition of cardinality
- If $A \cap B = \emptyset$, then $|A \cup B| = |A| + |B|$
- If $f : A \to B$ is one-to-one, then $|A| \leq |B|$.
- If $|A| > |B|$, and $f : A \to B$, then f is not one-to-one.

Ex. 6.3.3. If $A, B \subseteq X$ and f is one-to-one, then $F(A - B) \subseteq F(A) - F(B)$.

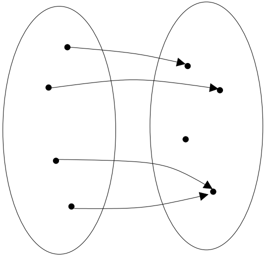
Ex. 6.4.1. If $f : X \to Y$, then $f \circ i_X = f$.



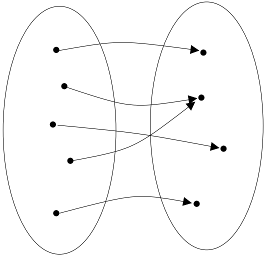
Not a function



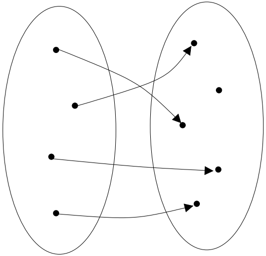
Not a function



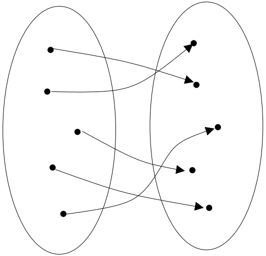
A function but not one-to-one or onto



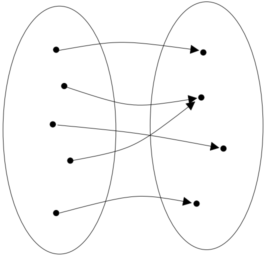
Onto, not one-to-one



One-to-one, not onto

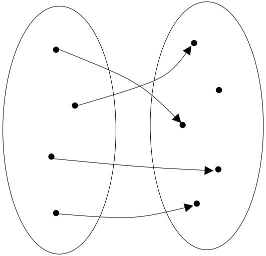


One-to-one correspondence



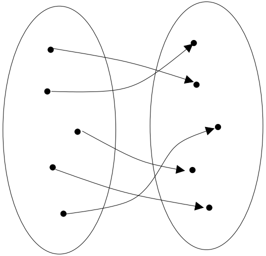
Onto, not one-to-one

$|X| \geq |Y|$



One-to-one, not onto

$|X| \leq |Y|$



One-to-one correspondence

$|X| = |Y|$

Two finite sets X and Y have the *the same cardinality* as each other if there exists a one-to-one correspondence from X to Y.

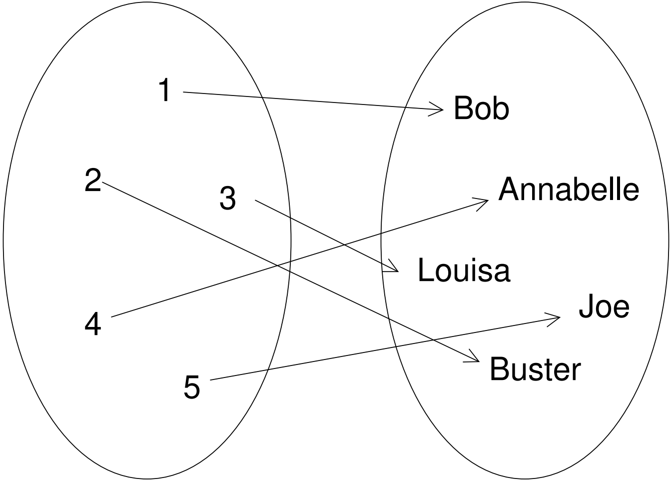
To use this *analytically*:

> Suppose X and Y have the same cardinality. Then let f be a one-to-one correspondence from X to Y.
> f is both onto and one-to-one.

To use this *synthetically*:

> *Given sets X and Y ...*
> **[Define f]** Let $f : X \to Y$ be a function defined as ...
> Suppose $y \in Y$. *Somehow find $x \in X$ such that $f(x) = y$.* Hence f is onto.
> Suppose $x_1, x_2 \in X$ such that $f(x_1) = f(x_2)$. *Somehow show $x_1 = x_2$.* Hence f is one-to-one.
> Since f is a one-to-one correspondence, X and Y have the same cardinality as each other.

A finite set X has cardinality $n \in \mathbb{N}$, which we write as $|X| = n$, if there exists a one-to-one correspondence from $\{1, 2, \ldots n\}$ to X. Moreover, $|\emptyset| = 0$.



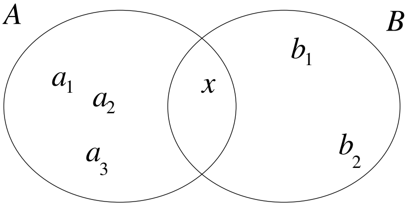
Theorem 6.12. If A and B are finite, disjoint sets, then $|A \cup B| = |A| + |B|$.

Theorem 6.13. If A and B are finite sets and $f : A \to B$ is one-to-one, then $|A| \leq |B|$.

Theorem 6.14. If A and B are finite sets, $|A| > |B|$, and $f : A \to B$, then f is not one-to-one.

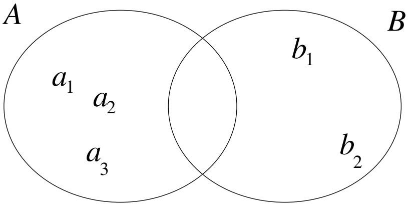
Exercise 6.6.5. If A and B are finite sets and $f : A \to B$ is onto, then $|A| \geq |B$. (Unassigned)

$A \cap B = \emptyset \quad \rightarrow \quad |A \cup B| = |A| + |B|$



$$|A \cup B| = |\{a_1, a_2, a_3, x, b_1, b_2\}| = 6$$

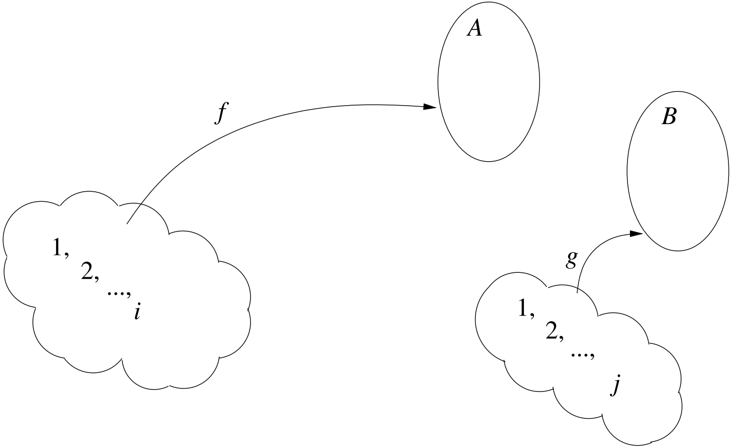
$$\begin{gathered} |A| + |B| = \\ = |\{a_1, a_2, a_3, x\}| + |\{x, b_1, b_2\}| \\ = 4 + 3 = 7 \end{gathered}$$



$$|A \cup B| = |\{a_1, a_2, a_3, b_1, b_2\}| = 5$$

$$\begin{gathered} |A| + |B| = \\ = |\{a_1, a_2, a_3\}| + |\{b_1, b_2\}| \\ = 3 + 2 = 5 \end{gathered}$$

$$A \cap B = \emptyset \quad \rightarrow \quad |A \cup B| = |A| + |B|$$



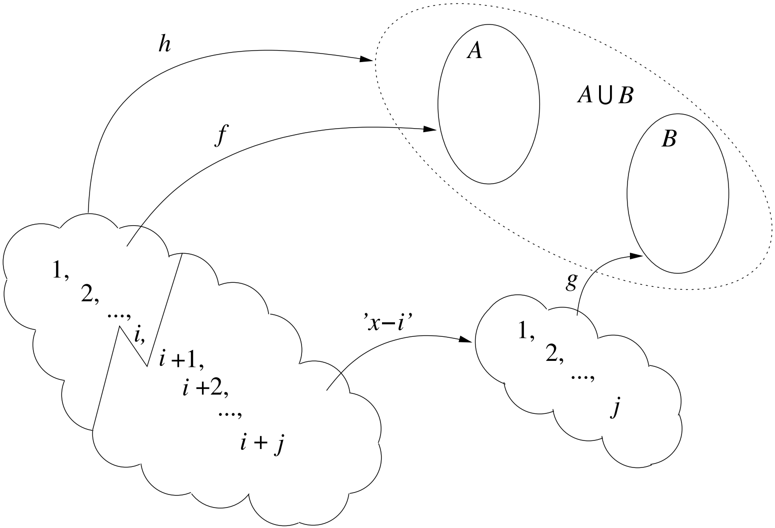
$$A \cap B = \emptyset \quad \rightarrow \quad |A \cup B| = |A| + |B|$$

x	f
1	Zed
2	Yelemis
3	Xavier

x	g
1	Wilhelmina
2	Valerie
3	Ursula
4	Tassie

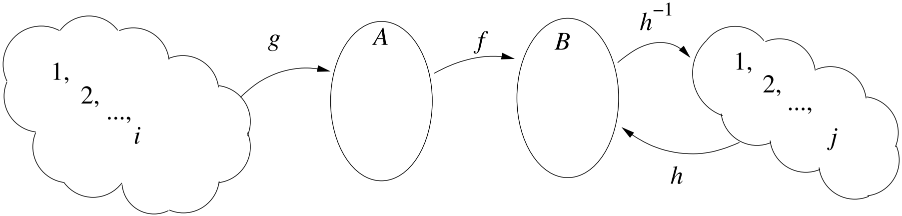
x	h				
1	$f(1)$	$=$	Zed		
2	$f(2)$	$=$	Yelemis		
3	$f(3)$	$=$	Xavier		
4	$g(4-3)$	$=$	$g(1)$	$=$	Wilhelmina
5	$g(5-3)$	$=$	$g(2)$	$=$	Valerie
6	$g(6-3)$	$=$	$g(3)$	$=$	Ursula
7	$g(7-3)$	$=$	$g(4)$	$=$	Tassie

$$A \cap B = \emptyset \quad \to \quad |A \cup B| = |A| + |B|$$

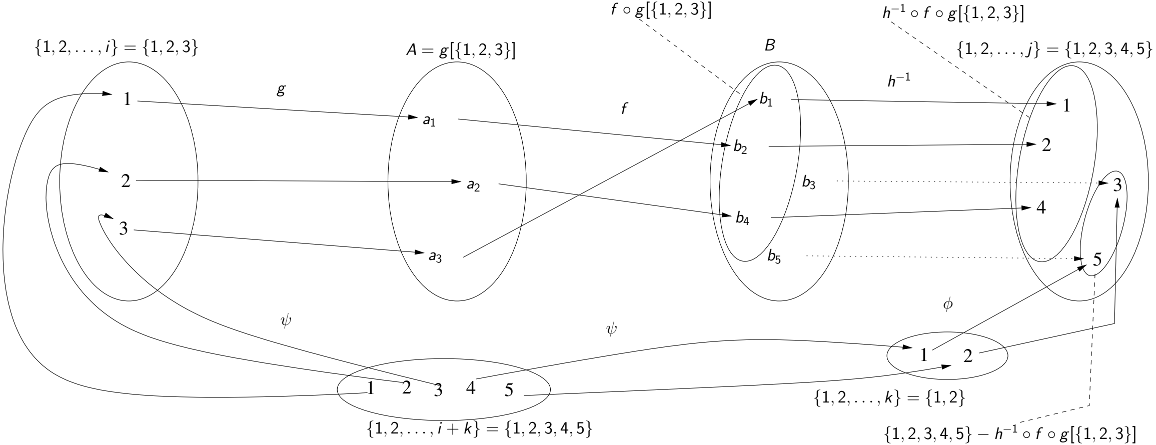


$f : A \to B$ is one-to-one $\to$ $|A| \leq |B|$

Theorem 6.13. If A and B are finite sets and $f : A \to B$ is one-to-one, then $|A| \leq |B|$.



$f : A \to B$ is one-to-one $\to$ $|A| \leq |B|$



Theorem 6.13. If A and B are finite sets and $f : A \to B$ is one-to-one, then $|A| \leq |B|$.

Theorem 6.14. If A and B are finite sets, $|A| > |B|$, and $f : A \to B$, then f is not one-to-one.

Art credit: Sharon Dunbar '23

For next time:

Do Exercises 6.6.(1 & 2). Both of these are simpler than they appear. See Canvas for hints/clarifications.

No reading or Canvas quiz
(But there is a Canvas quiz for next week Wednesday, Nov 19)