Chapter 3 roadmap:

- Propositions, booleans, logical equivalence. §3.1 (last week Friday)
- Boolean sequences §3.2 (Monday)
- Conditional propositions and arguments. §3.3 (Wednesday)
- Predicates and quantification. §3.(4 & 5) (**Today**)
- (Begin proofs next week)

Today:

- Predicates
  - Definition
  - Predicates in code
  - Anonymous functions
- Quantification
  - Universal quantification
  - Existential quantification
  - Quantification in code
  - Nested quantification

Four ways to interpret/define the idea of a *predicate*

- A predicate is a proposition with a parameter.

$$x < 5 \qquad x \text{ is orange}$$

- A predicate is a function whose value is true or false.

$$P(x) = x < 5 \qquad Q(x) = x \text{ is orange}$$

- A predicate is a part of a sentence that complements a noun phrase to make a proposition.
  A pumpkin is orange.
- A predicate is a truth set

| | | |
|---|---|---|
| | $P : \mathbb{N} \to \mathbb{B}, P(x) = x < 5$ | $Q(x) = x$ is orange |
| Truth set: | $\{1, 2, 3, 4\}$ | { pumpkin, fall leaves, orange juice, ... } |

## Universal quantification

"For all multiples of 3, the sum of their digits is a multiple of 3."

Let $D$ be the set of multiples of 3, that is
$D = \{n \in \mathbb{N} \mid n \bmod 3 = 0\} = \{3, 6, 9, 12, 15, 18, \ldots\}$

$$\forall\, x \in D, \texttt{sum}(\texttt{digify}(x)) \in D$$

Other examples:

- $\forall x \in \{5, 7, 19, 23, 43\}$, $x$ is prime.
- $\forall x \in \{4, 16, 25, 31\}$, $x$ is a perfect square.

Existential quantification

"There is a multiple of 3 that is not a perfect square."

$$\exists\ x \in D \mid x \text{ is not a perfect square}$$

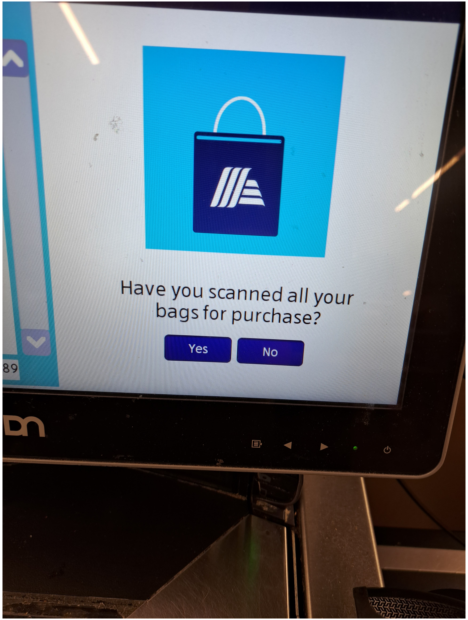
Alternately, "Some multiples of 3 are not perfect squares."

General forms for universal and existential quantification:

$$\forall\ x \in X, P(x) \qquad\qquad \exists\ x \in X \mid P(x)$$

$\forall\ x \in \emptyset, P(x)$ is **always (vacuously) true**.

$\exists\ x \in \emptyset \mid P(x)$ is **always false**



Have you scanned all your
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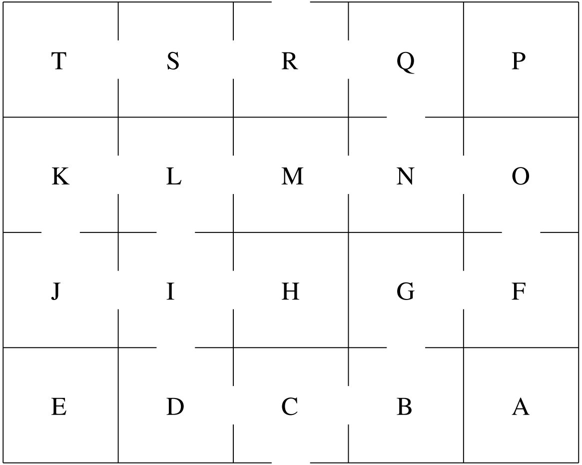
Yes
No
89

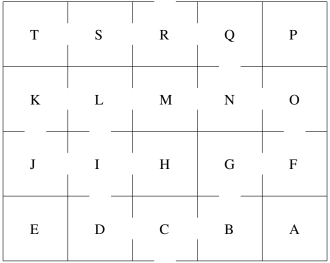
$$
\begin{aligned}
&\sim (\forall\, x \in X, P(x)) \\
&\quad \equiv \quad \sim (P(x_1) \wedge P(x_2) \wedge \cdots) \\
&\quad \equiv \quad \sim P(x_1) \vee \sim P(x_2) \vee \cdots \quad \text{By DeMorgan's Law} \\
&\quad \equiv \quad \exists\, x \in X \mid\ \sim P(x)
\end{aligned}
$$




1. Bob passed through $P$.
2. Bob passed through $N$.
3. Bob passed through $M$.
4. If Bob passed through $O$, then Bob passed through $F$.
5. If Bob passed through $K$, then Bob passed through $L$.
6. If Bob passed through $L$, then Bob passed through $K$.

Based on example by Susanna Epp, 2006

Let $X$ be the routes through the maze, that is,
$X = \{CBGFONQR, CDILMNQR, CDIJKLMNQR\}$




Let $P(x)$ = route $x$ contains $L$,
$Q(x)$ = route $x$ contains $K$.

Consider $\forall\, x \in X, P(x) \to Q(x)$.

| $X$ | $P(x)$ | $Q(x)$ | $P(x) \to Q(x)$ |
|---|---|---|---|
| $CBGFONQR$ | | | |
| $CDILMNQR$ | | | |
| $CDIJKLMNQR$ | | | |

**For next time:**

*Do Exercises 3.4.(2, 5-9) and 3.5.(22-26).*
*(All programming)*

*Read Section 4.1*

*Take quiz*