Chapter 3:

- Propositions, booleans, logical equivalence. §3.1 (**Today**)
- Boolean sequences §3.2 (next week Monday)
- Conditional propositions and arguments. §3.3 (next week Wednesday)
- Predicates and quantification. §3.(4 & 5) (next week Friday)
- (Begin proofs next week after)

Today:

- Highlight points from the beginning of §3.1: Propositions, forms, etc
- Take a programming break
- Work through the latter part of §3.1: Logical equivalences (Game 1)

A **proposition** is a sentence that is true or false, but not both.

It is snowing and it is not Thursday.

A **propositional form** is like a proposition but with content replaced by variables.

p and not q

$p \wedge \sim q$

$$\mathbb{Z} = \{\ldots -3, -2, -1, 0, 1, 2, 3 \ldots\}$$

$$\mathbb{B} = \{T, F\}$$

$$+ - \times \div$$

$$\vee \wedge \sim$$

$\times$	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	4	6
3	0	3	6	9

$\wedge$	T	F
T	T	F
F	F	F

$\wedge$	T	F
T	T	F
F	F	F

$\vee$	T	F
T	T	T
F	T	F

p	$\sim p$
T	F
F	T

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

p	q	$p \wedge q$	$p \vee q$	$\sim p$
T	T	T	T	F
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

Evaluate (to T or F) this logical expression:

$(T \wedge (\sim F \vee F)) \wedge (T \wedge T)$

Evaluate (to T or F) this logical expression:

$$(T \vee F) \wedge \sim (F \wedge T)$$

Evaluate (to T or F) this logical expression:

$(F \vee F \vee T) \wedge (\sim T \wedge F)$

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim (p \wedge q)$	$\sim p \vee \sim q$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

Commutativity	*Addition*	$x + y = y + x$
	Multiplication	$x \cdot y = y \cdot x$
Associativity	*Addition*	$(x + y) + z = x + (y + z)$
	Multiplication	$(x \cdot y) \cdot z = x \cdot (y \cdot z)$
Identity	*Addition*	$x + 0 = x$
	Multiplication	$x \cdot 1 = x$
Universal bounds	*Multiplication*	$x \cdot 0 = 0$

Commutativity	*Union*	$A \cup B = B \cup A$
	Intersection	$A \cap B = B \cap A$
	Symmetric difference	$A \oplus B = B \oplus A$
Associativity	*Union*	$(A \cup B) \cup C = A \cup (B \cup C)$
	Intersection	$(A \cap B) \cap C = A \cap (B \cap C)$
	Symmetric difference	$(A \oplus B) \oplus C = A \oplus (B \oplus C)$
Identity	*Union*	$A \cup \emptyset = A$
	Intersection	$A \cap \mathcal{U} = A$
Universal bounds	*Union*	$A \cup \mathcal{U} = \mathcal{U}$
	Intersection	$A \cap \emptyset = \emptyset$

Commutativity	*Conjunction*	$p \wedge q \equiv q \wedge p$
	Disjunction	$p \vee q \equiv q \vee p$
Associativity	*Conjunction*	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
	Disjunction	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
Identity	*Conjunction*	$p \wedge T \equiv p$
	Disjunction	$p \vee F \equiv p$
Universal bounds	*Conjunction*	$p \wedge F \equiv F$
	Disjunction	$p \vee T \equiv T$

Commutativity:	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
Associativity:	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
Distributivity:	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
Absorption:	$p \wedge (p \vee q) \equiv p$	$p \vee (p \wedge q) \equiv p$
Idempotency:	$p \wedge p \equiv p$	$p \vee p \equiv p$
Double negation:	$\sim\sim p \equiv p$	
DeMorgan's:	$\sim (p \wedge q) \equiv \sim p \vee \sim q$	$\sim (p \vee q) \equiv \sim p \wedge \sim q$
Negation:	$p \vee \sim p \equiv T$	$p \wedge \sim p \equiv F$
Universal bounds:	$p \vee T \equiv T$	$p \wedge F \equiv F$
Identity:	$p \wedge T \equiv p$	$p \vee F \equiv p$
Tautology and contradiction:	$\sim T \equiv F$	$\sim F \equiv T$

Remember from high school algebra that there are "simplify" problems and "solve" problems.

- Simplify $3x(2+3x)^2+1$.

$$\begin{aligned} & 3x(2+3x)^2+1 \\ = \quad & 3x(4+12x+9x^2)+1 \\ = \quad & 12x+36x^2+27x^3+1 \\ = \quad & 27x^3+36x^2+12x+1 \end{aligned}$$

- Solve $12x = 57 - 7x$ for x.

$$\begin{aligned} 12x &= 57 - 7x \\ 19x &= 57 \\ x &= 3 \end{aligned}$$

Suppose we were to show that $\sim(\sim p \wedge q) \vee (p \vee \sim p) \equiv p \vee \sim q$.

Do this:

$$
\begin{array}{lll}
 & \sim(\sim p \wedge q) \vee (p \wedge \sim p) & \\
\equiv & \sim(\sim p \wedge q) \vee F & \text{by negation law} \\
\equiv & \sim(\sim p \wedge q) & \text{by identity law} \\
\equiv & p \vee \sim q & \text{by De Morgan's}
\end{array}
$$

Don't do this:

$$
\begin{array}{rcll}
\sim(\sim p \wedge q) \vee (p \wedge \sim p) & \equiv & p \vee \sim q & \\
\sim(\sim p \wedge q) \vee F & \equiv & p \vee \sim q & \text{by negation law} \\
\sim(\sim p \wedge q) & \equiv & p \vee \sim q & \text{by identity law} \\
p \vee \sim q & \equiv & p \vee \sim q & \text{by De Morgan's}
\end{array}
$$

For next time:

Do Exercises 3.1.(19, 21, 25-29).

Read 3.1, if you haven't already
Read 3.2
Take quiz (covers both 3.1 and 3.2)