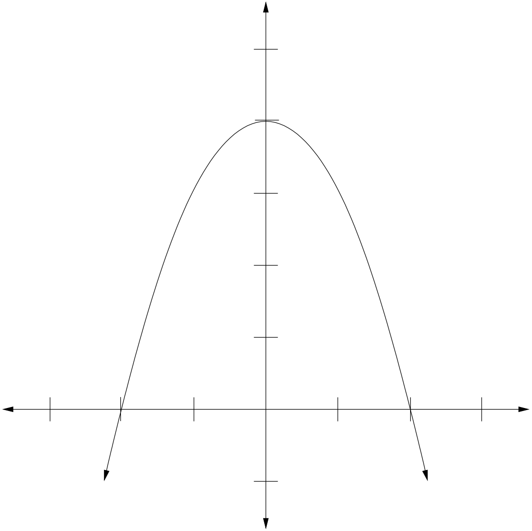
Chapter 5 roadmap:

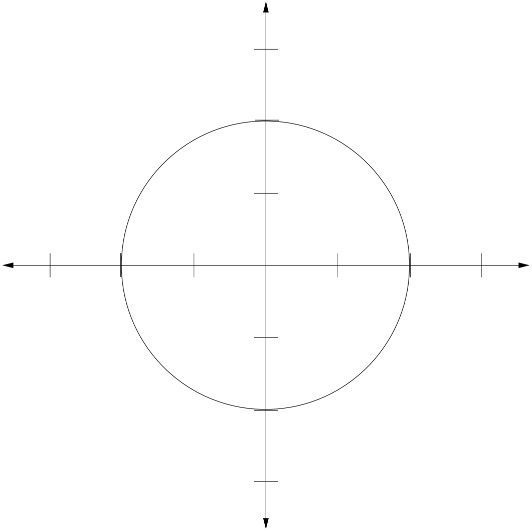
- Introduction to relations (**Today**)
- Images and composition; relation properties (Monday)
- Proofs about relations (Wednesday)
- Closures (Friday)
- Partial order relations (next week Monday)

Today: Introduction to relations

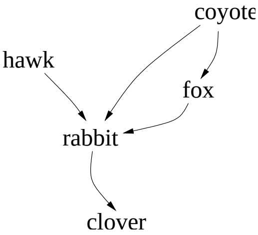
- Definition
- Examples
- Other terms
  - Image
  - Inverse
  - Composition
- Code representation
- Proofs



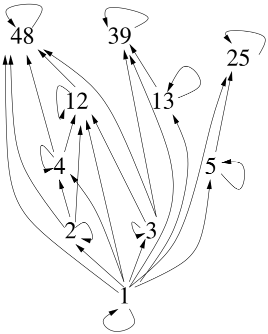
$y = 4 - x^2$



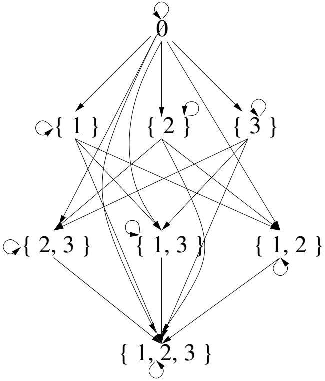
$y^2 = 4 - x^2$



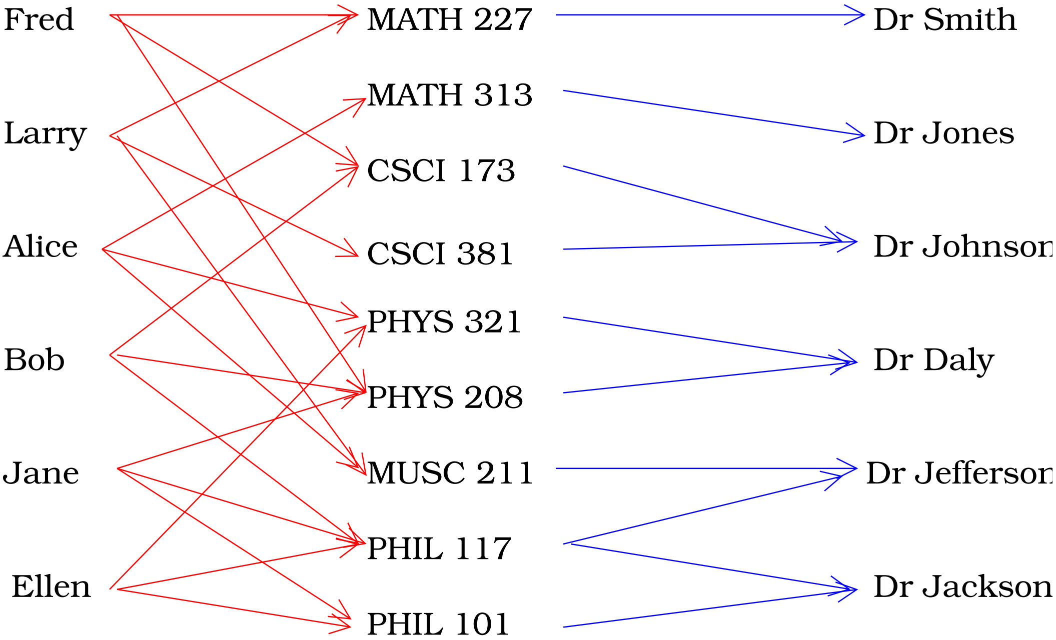
eats



| (divides)

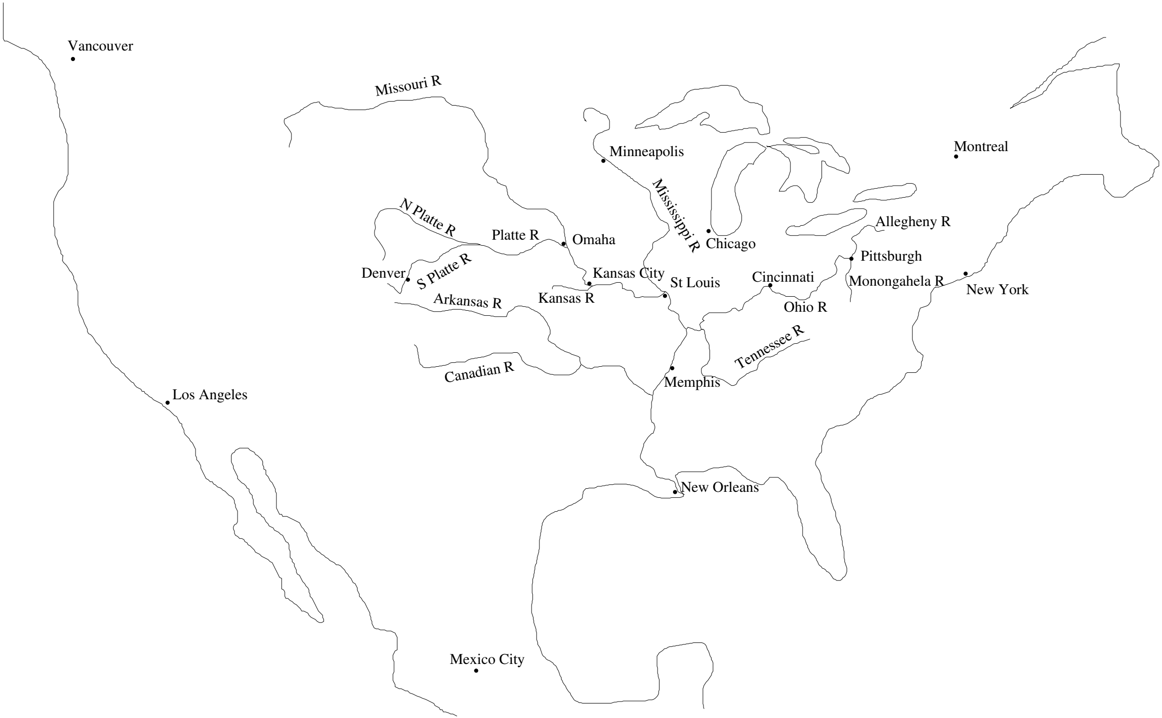


⊆ (subset)



Fred
Larry
Alice
Bob
Jane
Ellen
MATH 227
MATH 313
CSCI 173
CSCI 381
PHYS 321
PHYS 208
MUSC 211
PHIL 117
PHIL 101
Dr Smith
Dr Jones
Dr Johnson
Dr Daly
Dr Jefferson
Dr Jackson

A **relation** from one set to another	R	set of pairs	subset of $X \times Y$ $R \subseteq X \times Y$	isEnrolledIn, isTaughtBy
A **relation** on a set	R	set of pairs	subset of $X \times X$ $R \subseteq X \times X$	eats, divides
The **image** of an element under a relation	$\mathcal{I}_R(a)$	set	set of things that a is related to $\mathcal{I}_R(a) = \{b \in Y \mid (a, b) \in R\}$	classes Bob is enrolled in, numbers that 4 divides
The **image** of a set under a relation	$\mathcal{I}_R(A)$	set	set of things that things in A are related to $\mathcal{I}_R(A) = \{b \in Y \mid \exists\ a \in A \mid (a, b) \in R\}$	classes Bob, Larry, or Alice are taking, numbers that 2, 3, or 5 divide
The **inverse** of a relation	R^{-1}	relation	the arrows/pairs of R reversed $R^{-1} = \{(b, a) \in Y \times X \mid (a, b) \in R\}$	hasOnRoster, teaches, isEatenBy, isDivisibleBy
The **composition** of two relations	$S \circ R$	relation	two hops combined to one hop (Assume $S \subseteq Y \times Z$) $S \circ R = \{(a, c) \in X \times Z \mid \exists\ b \in Y \mid (a, b) \in R \wedge (b, c) \in S\}$	hasAsProfessor, eatsSomethingThatEats
The **identity** relation on a set	i_X	relation	everything is related only to itself $i_X = \{(x, x) \mid x \in X\}$	$=$


Vancouver
Missouri R
Minneapolis
Montreal
Mississippi R
N Platte R
Platte R
Omaha
Chicago
Allegheny R
Pittsburgh
Denver
S Platte R
Kansas City
St Louis
Cincinnati
Monongahela R
New York
Arkansas R
Kansas R
Ohio R
Tennessee R
Canadian R
Memphis
Los Angeles
New Orleans
Mexico City

Theorem 5.1 If $a, b \in \mathbb{N}$ and $a|b$, then $\mathcal{I}_|(b) \subseteq \mathcal{I}_|(a)$.

Theorem 5.2 If R is a relation on a set A, $a \in A$, and $\mathcal{I}_R(a) \neq \emptyset$, then $a \in \mathcal{I}_{R^{-1}}(\mathcal{I}_R(a))$.

Ex 5.2.7 Prove that if R is a relation over a set A and $(a, b) \in R$, then $\mathcal{I}_R(b) \subseteq \mathcal{I}_{R \circ R}(a)$.

Ex 5.2.8 Suppose R is a relation from a set X to a set Y and $A \subseteq X$. Are either of the following true?

$\mathcal{I}_{R^{-1}}(\mathcal{I}_R(A)) \subseteq A$.

$A \subseteq \mathcal{I}_{R^{-1}}(\mathcal{I}_R(A))$.

Prove or give a counterexample for each.

Ex 5.2.9 Prove that for a relation R from A to B, $i_B \circ R = R$.

Ex 5.2.10 Prove that if R is a relation from A to B, then $(R^{-1})^{-1} = R$.

Ex 5.2.11 If R is a relation from A to B, is $R^{-1} \circ R = i_A$? Prove or give a counterexample.

For next time:

Due **Monday, Oct 27***:*
Do Exercises 5.1.5 and 5.2.(7, 8, 12, 13, 14, 15)
See Canvas for hints/explanations.
Read Section 5.3
Take quiz on Section 5.3