Chapter 4 roadmap:

- Subset proofs (last week Monday)
- Set equality and emptiness proofs (last week Wednesday)
- Conditional and biconditional proofs (last week Friday)
- Proofs about powersets (Monday)
- Review for Test 2 (**today**)
- Test 2 (Friday)
- (Begin Chapter 5 Relations Friday after fall break)

Today:

- What to expect
- Tips about Game 1 and Game 2
- Tips about programming problems
- Review of some proof HW problems
- Hints on a powerset HW problem
- General review of Ch 3 & 4

Goals of this course

- Write programs in the functional style
- Think recursively
- Understand sets, relations, and functions so that they can model real-world (and abstract) information
- Use formal logic to prove mathematical propositions.

Concepts of Chapter 3 and 4

- The system of propositional logic, including logical equivalences and arguments
- Boolean operations and predicates in functional programming
- Quantification
- Proofs of set propositions
- Proofs of conditional propositions
- "Advanced proofs", e.g. with powersets

Test content organized by standards:

Standard 7. Verify logical equivalences using algebraic simplifications. *The test will include one "Game One" problem in which you verify a logical equivalence using known logical equivalences.*

Standard 8. Verify argument forms using previously known argument forms. *The test will include one "Game Two" problem in which you verify the validity of an argument form using known argument forms.*

Standard 9. Write Python functions using filter and anonymous functions. *The test will include two problems that ask you to write functions using `filter_set`, `filter_list` or `filter` (built-in) and anonymous functions (lambda expressions).*

Standard 10. Write Boolean-valued Python functions. *The test will include two problems that ask you to write a Boolean-valued function specified using quantification.*

Standard 11. Write proofs for basic set propositions [subset, set equality, set emptiness]. *The test will include two problems that ask you to write a proof of a "basic" set proposition involving subset, set equality, or set emptiness.*

Standard 12. Write proofs for advanced set propositions [conditionals, biconditionals, powersets]. *The test will include two problems that ask you to write a proof of an "advanced" set proposition involving a conditional or biconditional and/or powersets.*

Which of the following are true?

$$
\begin{aligned}
-((x-y)+(x-z)) &= -(x-y)-(x-z) \\
-((x-y)+(x-z))\cdot z &= -(x-y)-(x-z)\cdot z \\
\sim(p \wedge q) &\equiv \sim p \vee \sim q \\
\sim(p \wedge q) \wedge r &\equiv \sim p \vee \sim q \wedge r
\end{aligned}
$$

Which of the following are true?

$$\begin{aligned}
(x+y)+z &= x+(y+z) \\
(x-y)+z &= x-(y+z) \\
(p \vee q) \vee r &\equiv p \vee (q \vee r) \\
(p \vee q) \wedge r &\equiv p \vee (q \wedge r)
\end{aligned}$$

$$
\begin{aligned}
& ((q \wedge (p \wedge (p \vee q))) \vee (q \wedge \sim p)) \wedge \sim q & \\
\equiv \quad & ((q \wedge p) \vee (q \wedge \sim p)) \wedge \sim q & \text{Absorption} \\
\equiv \quad & (q \wedge (p \vee \sim p)) \wedge \sim q & \text{Distributivity} \\
\equiv \quad & (q \wedge T) \wedge \sim q & \text{Negation} \\
\equiv \quad & q \wedge \sim q & \text{Identity} \\
\equiv \quad & F & \text{Negation}
\end{aligned}
$$

WRONG!

$$((q \wedge (p \wedge (p \vee q))) \vee (q \wedge \sim p)) \wedge \sim q$$

$$\equiv \quad ((q \wedge p) \vee (q \wedge \sim p)) \wedge \sim q \qquad \text{Absorption}$$

$$\equiv \quad (q \wedge p) \vee ((q \wedge \sim p) \wedge \sim q) \qquad \text{Associativity}$$

Suppose we were to show that $\sim(\sim p \wedge q) \vee (p \vee \sim p) \equiv p \vee \sim q$.

Do this:

$$\begin{aligned} & \sim(\sim p \wedge q) \vee (p \wedge \sim p) & \\ \equiv\ & \sim(\sim p \wedge q) \vee F & \text{by negation law} \\ \equiv\ & \sim(\sim p \wedge q) & \text{by identity law} \\ \equiv\ & p \vee \sim q & \text{by De Morgan's} \end{aligned}$$

Don't do this:

$$\begin{aligned} \sim(\sim p \wedge q) \vee (p \wedge \sim p) &\equiv p \vee \sim q & \\ \sim(\sim p \wedge q) \vee F &\equiv p \vee \sim q & \text{by negation law} \\ \sim(\sim p \wedge q) &\equiv p \vee \sim q & \text{by identity law} \\ p \vee \sim q &\equiv p \vee \sim q & \text{by De Morgan's} \end{aligned}$$

Modus Ponens

$p \to q$

p

$\therefore q$

Modus Tollens

$p \to q$

$\sim q$

$\therefore \sim p$

Generalization

p

$\therefore p \vee q$

Specialization

$p \wedge q$

$\therefore p$

Elimination

$p \vee q$

$\sim p$

$\therefore q$

Transitivity

$p \to q$

$q \to r$

$\therefore p \to r$

Division into cases

$p \vee q$

$p \to r$

$q \to r$

$\therefore r$

Contradiction

$p \to F$

$\therefore \sim p$

For next time:

Study for test...

Read Sections 5.(1 & 2) for Friday, Oct 24
Take quiz