Chapter 6 outline:

- Introduction, function equality, and anonymous functions (week-before Wednesday)
- Image and inverse images (week-before Friday)
- Function properties and composition (last week Monday)
- Map, filter, reduce, and pipelines (last week Wednesday)
- Cardinality (last week Friday)
- Countability (Monday)
- Review (**today**)
- Test 3, on Ch 5 & 6 (Friday)
- (Begin self-reference chapter next week Monday)

Today:

- What to expect for "relations" questions
- What to expect for programming questions
- What to expect for "functions" questions
- How can I help you?

Goals of this course

- Write programs in the functional style
- Think recursively
- Understand sets, relations, and functions so that they can model real-world (and abstract) information
- Use formal logic to prove mathematical propositions.

Concepts of Chapters 5 & 6

- What functions, relations, their properties, and their related terms mean
- How to apply formal definitions in proofs
- Modeling relations and functions in programs and applying those models to solve problems

Concepts

5.(1–3). The definitions of *relation*, *image*, *inverse*, *identity relation*, and *composition*. *reflexive*, *symmetric*, and/or *transitive*.

Standards

Standard 13. Write proofs for propositions about relations and their properties.

If R is a symmetric relation on X, then $R - i_X$ is symmetric.

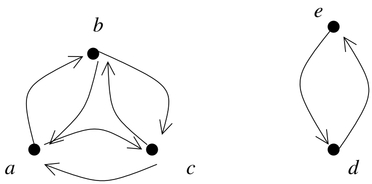
- A *relation* R from a set X to a set Y is a set of ordered pairs from X and Y; it is a subset of $X \times Y$.
- The *identity* relation on a set is $i_X = \{(x, x) \mid x \in X\}$.
- A relation R is *symmetric* if $\forall\ (x, y) \in R$, $(y, x) \in R$.

Concepts

5.(4 & 5). The definitions of *reflexive*, *symmetric*, *transitive*, *antisymmetric*, *transitive (and other) closure*, *partial order relation*, *total order relation*, and *topological sort*.

Standards

Standard 14. Identify relation properties and derived relations such as transitive closure, topological sort.



a. Is this relation reflexive?
If **no**, then give a counterexample.

b. . . . symmetric?

c. . . . transitive?

d. . . . antisymmetric?

e. Draw the transitive closure of this relation.

f. Give a topological sort for the transitive closure of this relation or explain why one does not exist.

Concepts

6.1. Finite functions represented as Python dictionaries; dictionary subscripting, dictionary inclusion (`in`), dictionary comprehensions, dictionary union (`&`).

6.5. The `reduce` function as a means of applying an operation sequentially over items in a collection.

Standards

Standard 16. Write Python functions that process dictionaries.

Standard 15. Write Python functions that use reduce and related tools.

Concepts

6.(1–4). The definitions of *function*, *function equality*, *image inverse image*, *one-to-one*, *onto*, and *composition*.

Standards

Standard 17. Write proofs for basic function propositions (image, inverse image, function properties).

Standard 18. Write proofs for advanced function propositions (interactions among image, inverse image, function properties, and composition)

If $f : X \to Y$, $A \subseteq Y$, and f is onto, then $A \subseteq f[f^{-1}[A]]$.

- If f is a function from a set X to a set Y and $A \subseteq X$, then the *image* of A under f is $f[A] = \{y \in Y \mid \exists\ a \in A \text{ such that } f(a) = y\}$
- ...and $B \subseteq Y$, then the *inverse image* of B under f is $f^{-1}[B] = \{x \in X \mid f(x) \in B\}$
- f is onto if $\forall\ y \in\ Y, \exists\ x \in X \mid f(x) = y$.

Concepts

6.(1–4). The definitions of *function*, *function equality*, *image inverse image*, *one-to-one*, *onto*, and *composition*.

Standards

Standard 17. Write proofs for basic function propositions (image, inverse image, function properties).

Standard 18. Write proofs for advanced function propositions (interactions among image, inverse image, function properties, and composition)

If $f : X \to Y$ and $g : Y \to Z$ are both one-to-one, then $g \circ f : X \to Z$ is one-to-one.

- If f is a function from a set X to a set Y, then f is *one-to-one* if $\forall\ x_1, x_2 \in X$, if $f(x_1) = f(x_2)$ then $x_1 = x_2$.
- f $f : X \to Y$ and $g : Y \to Z$, then the *composition* of f and g is the function $g \circ f = \{(x, z) \in X \times Z \mid z = g(f(x))\}$.

For next time:

Study for test...

There will be a section to read for Monday (first section of Chapter 7)