Chapter 1 outline:

- Introduction, sets and elements (week-before Wednesday)
- Python expressions (week-before Friday)
- Python functions; denoting sets (last week Wednesday)
- Set operations; visual verification of set propositions (last week Friday)
- Cardinality, Cartesian products, powersets (**today**)
- (Begin Chapter 2 Sequence next time)

Today:

- Cardinality
- Disjoint[edness], pairwise disjoint[edness], partitions
- Cartesian products
- Powersets

term	**grammar**	**informal definition**	**formal definition**
Cardinality	noun	The cardinality of a set is the number of elements in that set.	See Chapter 6.
Disjoint	adjective	Two sets are disjoint if they have no elements in common.	X and Y are disjoint if $X \cap Y = \emptyset$.
Pairwise disjoint	adjective	A collection of sets are pairwise disjoint if no two of them have any elements in common.	The sets $X_1, X_2, \ldots, X_n$ are pairwise disjoint if for any two sets X_i and X_j, where $i \neq j$, $X_i \cap X_j = \emptyset$.
Partition	noun	A collection of subsets of a set are a partition of that set if they are pairwise disjoint and together make up the entire set.	If X is a set, then a partition of X is a set of sets $\{X_1, X_2, \ldots, X_n\}$ such that $X_1, X_2, \ldots, X_n$ are pairwise disjoint and $X_1 \cup X_2 \cup \ldots \cup X_n = X$.

Compute the cardinality:

$|\{1, 2, 3, 4, 5\} \cup \{3, 4, 5, 6\}|$

$|[0, \pi) \cap \mathbb{Z}|$

$\{0, 1, 2, \ldots n\}$

$|FacultyInThisRoom - StudentsInThisRoom|$

Which are disjoint?

$\mathbb{Z}$ and $\mathbb{R}$

$\mathbb{Z}$ and $\mathbb{R}^-$

$[0, 5)$ and $[5, 10)$

Plants and Fungi

$\mathit{MathClasses}$ and $\mathit{CSCIClasses}$

$\mathit{DeciduousTrees}$ and $\mathit{ConiferousTrees}$

1.7.5 One might be tempted to think $|A \cup B| = |A| + |B|$, but this is not true in general. Why not? (Assume A and B are finite.)

1.7.6 Is $|A - B| = |A| - |B|$ true in general? If so, explain why. If not, under what special circumstances is it true? (Assume A and B are finite.)

1.7.7 Consider the sets $\{1, 2, 3\}$, $\{2, 3, 4\}$, $\{3, 4, 5\}$, and $\{4, 5, 6\}$. Notice that

$$\{1, 2, 3\} \cap \{2, 3, 4\} \cap \{3, 4, 5\} \cap \{4, 5, 6\} = \emptyset$$

Are these sets pairwise disjoint?

1.7.8 Section 1.7 mentions two partitions of $\mathbb{Z}$:

$\{\{x \in \mathbb{Z} \mid x \mod 2 = 0\}, \{x \in \mathbb{Z} \mid x \mod 2 = 0\}\}$ $\{\mathbb{Z}^-, \{0\}, \mathbb{Z}^+\}$

Come up with another partition of $\mathbb{Z}$.

Describe the following Cartesian product: $\{-1, 0, 1\} \times \{a, b\}$

1.7.12 If A and B are finite sets, what is $|A \times B|$ in terms of $|A|$ and $|B|$?

1.7.13 Based on our description of the real number plane as a Cartesian product, explain how a line can be interpreted as a set.

1.7.14 Explain how $\mathbb{C}$, the set of complex numbers, can be thought of as a Cartesian product.

1.7.15 Any rational number (an element of set $\mathbb{Q}$) has two integers as components. Why not rewrite fractions as ordered pairs (for example, $\frac{1}{2}$ as $(1, 2)$ and $\frac{3}{4}$ as $(3, 4)$) and claim that $\mathbb{Q}$ can be thought of as $\mathbb{Z} \times \mathbb{Z}$? Explain why these two sets *cannot* be thought of as two different ways to write the same set. (There are at least two reasons.)

Which are true?

$\{3\} \in \mathscr{P}(\{1,2,3,4,5\})$

$3 \in \mathscr{P}(\{1,2,3,4,5\})$

$\{3\} \subseteq \mathscr{P}(\{1,2,3,4,5\})$

$3 \subseteq \mathscr{P}(\{1,2,3,4,5\})$

$a \in A$ iff $\{a\} \in \mathscr{P}(A)$

$a \in A$ iff $\{a\} \subseteq \mathscr{P}(A)$

$A \subseteq B$ iff $A \subseteq \mathscr{P}(B)$

$A \subseteq B$ iff $A \in \mathscr{P}(B)$

Which are true?

$\{A\} \subseteq \mathscr{P}(A)$

$A \in \mathscr{P}(A)$

$\{A\} \in \mathscr{P}(A)$

$\mathbb{Z} \in \mathscr{P}(\mathbb{R})$

$\emptyset \in \mathscr{P}(A)$

$\emptyset = \mathscr{P}(\emptyset)$

Note that

- $a \in A$ iff $\{a\} \in \mathscr{P}(A)$
- $A \subseteq B$ iff $A \in \mathscr{P}(B)$
- $A \subseteq B$ iff $\mathscr{P}(A) \subseteq \mathscr{P}(B)$
- $\mathscr{P}(\emptyset) = \{\emptyset\} \neq \emptyset$

Observe

$$\mathscr{P}(\{1,2,3\}) = \{ \emptyset, \{1\},\{2\},\{3\}, \{1,2\},\{1,3\},\{2,3\}, \{1,2,3\} \} = \{ \{1\},\{1,2\},\{1,3\},\{1,2,3\}, \emptyset,\{2\},\{3\},\{2,3\} \}$$

$$= \mathscr{P}(\{2,3\}) \cup \begin{bmatrix}\text{1 added to each set} \\ \text{of } \mathscr{P}(\{2,3\})\end{bmatrix} = \mathscr{P}(\{2,3\}) \cup \{ \{1\} \cup X \mid X \in \mathscr{P}(\{2,3\}) \}$$

If $a \in A$, then $\mathscr{P}(A) = \mathscr{P}(A - \{a\}) \cup \{ \{a\} \cup X \mid X \in \mathscr{P}(A - \{a\}) \}$

What is $|\mathscr{P}(X)|$ in terms of $|X|$?

For next time:

Pg 48–50: 1.7.(2, 3, 4, 11, 20, 21, 23)
Pg 43: 1.8.(2, 11, 14)

Read 2.1

Take quiz