Chapter 1 outline:

- Introduction, sets and elements (last week Wednesday)
- Python expressions (last week Friday)
- Python functions; denoting sets (Wednesday)
- Set operations; visual verification of set propositions (**today**)
- Cardinality, Cartesian products, powersets (next week Monday)
- (Begin Chapter 2 Sequence next week Wednesday)

Today:

- Set operations
  - Definitions
  - Python representation
  - Properties; analogy with arithmetic
- Verifying propositions visually

$\{1,5,7\} \not\subseteq \{1,7\}$ $\{1,5,7\} \not\subseteq \{1,4,7\}$ $\{1,5,7\} \subseteq \{1,5,7\}$ $\{1,5,7\} \subseteq \{1,4,5,7\}$

$\{1,5,7\} \not\subset \{1,7\}$ $\{1,5,7\} \not\subset \{1,4,7\}$ $\{1,5,7\} \not\subset \{1,5,7\}$ $\{1,5,7\} \subset \{1,4,5,7\}$

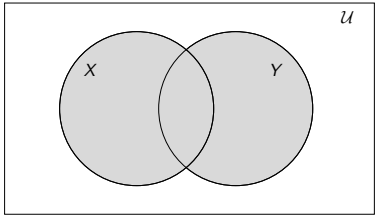
$\{1,5,7\} \neq \{1,7\}$ $\{1,5,7\} \neq \{1,4,7\}$ $\{1,5,7\} = \{1,5,7\}$ $\{1,5,7\} \neq \{1,4,5,7\}$

$\{1,5,7\} \supseteq \{1,7\}$ $\{1,5,7\} \not\supseteq \{1,4,7\}$ $\{1,5,7\} \supseteq \{1,5,7\}$ $\{1,5,7\} \not\supseteq \{1,4,5,7\}$

$\{1,5,7\} \supset \{1,7\}$ $\{1,5,7\} \not\supset \{1,4,7\}$ $\{1,5,7\} \not\supset \{1,5,7\}$ $\{1,5,7\} \not\supset \{1,4,5,7\}$

**Union** The set of elements that are in either set

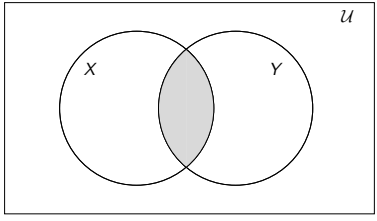
$$X \cup Y = \{x \mid x \in X \text{ or } x \in Y\}$$




$$\begin{aligned} \{1,2,3\} \cup \{2,3,4\} &= \{1,2,3,4\} \\ \{1,2\} \cup \{3,4\} &= \{1,2,3,4\} \\ \{1,2\} \cup \{1,2,3\} &= \{1,2,3\} \end{aligned}$$

**Intersection.** The set of elements that are in both sets

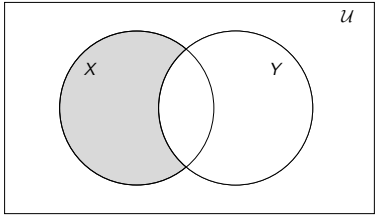
$$X \cap Y = \{x \mid X \in X \text{ and } x \in Y\}$$




$$\begin{aligned} \{1,2,3\} \cap \{2,3,4\} &= \{2,3\} \\ \{1,2\} \cap \{3,4\} &= \emptyset \\ \{1,2\} \cap \{1,2,3\} &= \{1,2\} \end{aligned}$$

**Difference.** The set of elements that are in the first set and not the other.

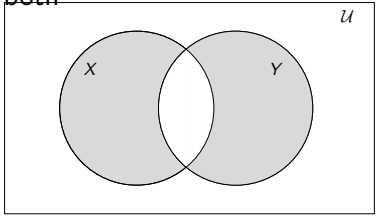
$$X - Y = \{x \mid x \in X \text{ and } x \notin Y\}$$




$$\begin{aligned} \{1,2,3\} - \{2,3,4\} &= \{1\} \\ \{1,2\} - \{3,4\} &= \{1,2\} \\ \{1,2\} - \{1,2,3\} &= \emptyset \end{aligned}$$

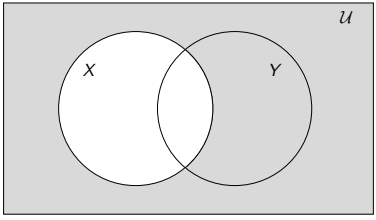
**Symmetric difference.** The set of elements that are in either set but not both

$$X \oplus Y = \{x \mid x \in X \cup Y \text{ and } x \notin X \cap Y\}$$




$$\begin{aligned} \{1,2,3\} \oplus \{2,3,4\} &= \{1,4\} \\ \{1,2\} \oplus \{3,4\} &= \{1,2,3,4\} \\ \{1,2\} \oplus \{1,2,3\} &= \{3\} \end{aligned}$$

**Complement.** The set of elements that are not in $X$




$$\overline{X} = \{x \mid x \in \mathcal{U} \text{ and } x \notin X\}$$

Let $\mathcal{U} = \{0, 1, 2, 3, 4\}$

$$\begin{aligned} \overline{\{1, 2, 3\}} &= \{0, 4\} \\ \overline{\{0, 2, 4\}} &= \{1, 3\} \\ \overline{\{0, 1, 2, 3, 4\}} &= \emptyset \end{aligned}$$

1. $\{1,2,3,4,5\} \cup \{5,6,7\} =$

2. $\{1,2,3,4,5\} \cap \{2,4,6,8,10\} =$

3. $\{1,2,3,4,5\} - \{2,3,4\} =$

4. $\{1,2,3,4,5\} - \{3,4,5,6,7\} =$

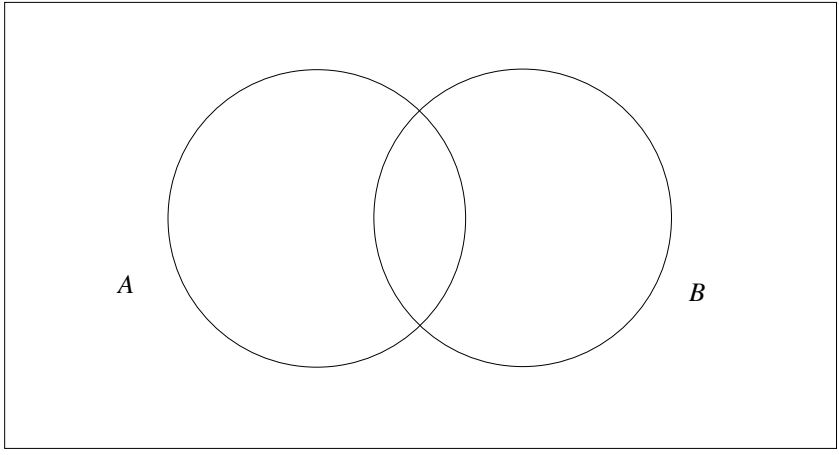
5. $\{1,2,3,4,5\} \oplus \{3,4,5,6,7\} =$

Which of the following are equal to $\{1, 2, 3, 4\}$ ?

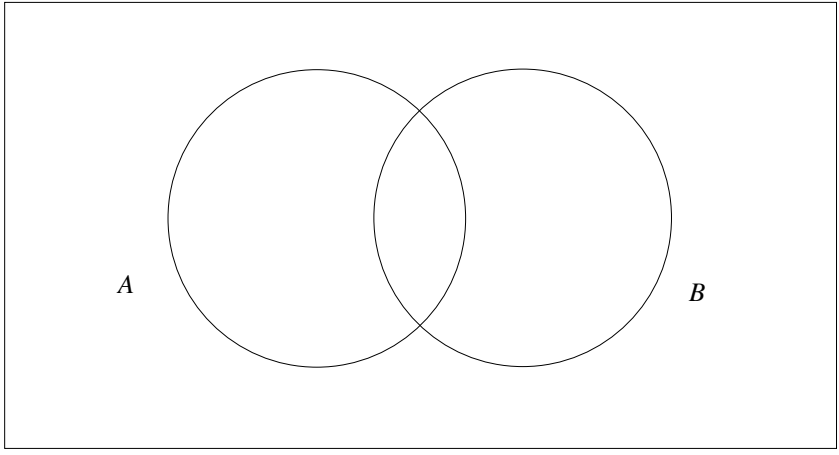
- $\{1, 2\} \cup \{3, 4\}$
- $\{1, 2, 3\} \cup \{4\}$
- $\{1, 2, 3\} \cup \{2, 3, 4\}$
- $\{1, 2, 3\} \cup \{3, 4, 5\}$
- $\{2, 3\} \cup \{1, 4\}$
- $\{1\} \cup \{3, 4\}$
- $\{4, 3, 2, 1\}$
- $\{1\} \cup \{1, 2\} \cup \{1, 2, 3\} \cup \{1, 2, 3, 4\}$

| | | |
|---|---|---|
| **Commutativity** | *Union* | $A \cup B = B \cup A$ |
| | *Intersection* | $A \cap B = B \cap A$ |
| | *Symmetric difference* | $A \oplus B = B \oplus A$ |
| **Associativity** | *Union* | $(A \cup B) \cup C = A \cup (B \cup C)$ |
| | *Intersection* | $(A \cap B) \cap C = A \cap (B \cap C)$ |
| | *Symmetric difference* | $(A \oplus B) \oplus C = A \oplus (B \oplus C)$ |
| **Identity** | *Union* | $A \cup \emptyset = A$ |
| | *Intersection* | $A \cap \mathcal{U} = A$ |
| **Universal bounds** | *Union* | $A \cup \mathcal{U} = \mathcal{U}$ |
| | *Intersection* | $A \cap \emptyset = \emptyset$ |

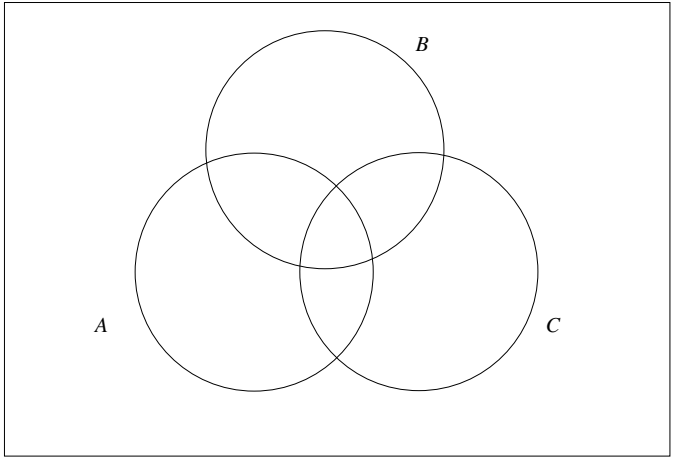
| | | |
|---|---|---|
| **Commutativity** | *Addition* | $x + y = y + x$ |
| | *Multiplication* | $x \cdot y = y \cdot x$ |
| **Associativity** | *Addition* | $(x + y) + z = x + (y + z)$ |
| | *Multiplication* | $(x \cdot y) \cdot z = x \cdot (y \cdot z)$ |
| **Identity** | *Addition* | $x + 0 = x$ |
| | *Multiplication* | $x \cdot 1 = x$ |
| **Universal bounds** | *Multiplication* | $x \cdot 0 = 0$ |



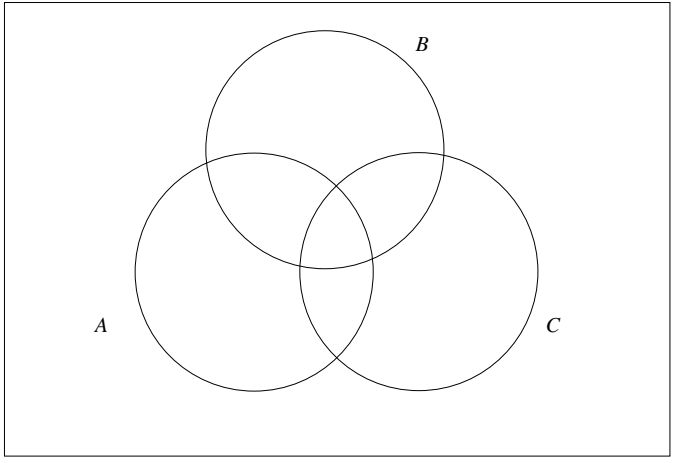

$(A \cap B) - A$



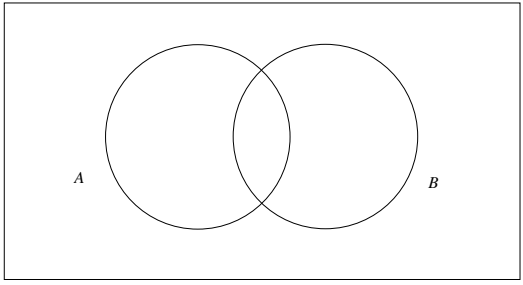

$(A - B) \cup (B - A)$

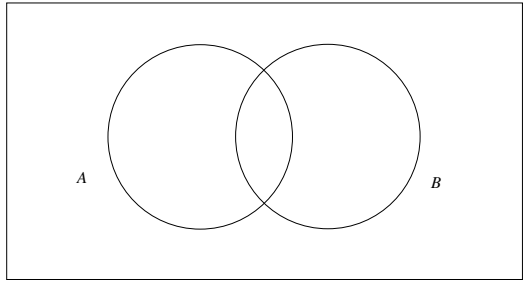



$(A \cup B) \cap (A \cup C)$

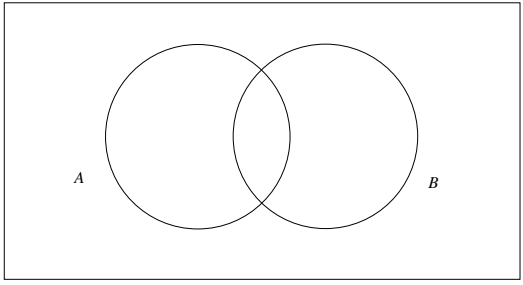


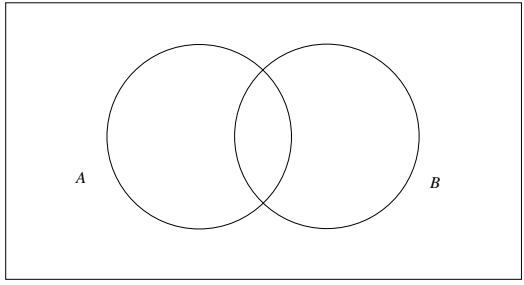

$\overline{(A \cap B)} \cap (A \cup C)$



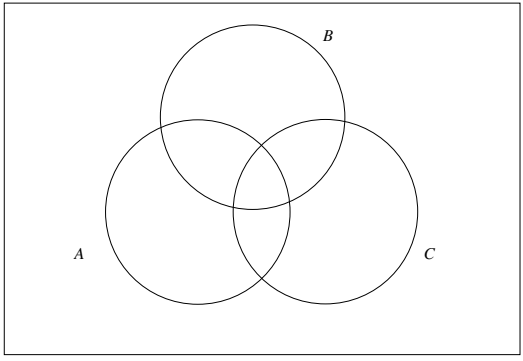



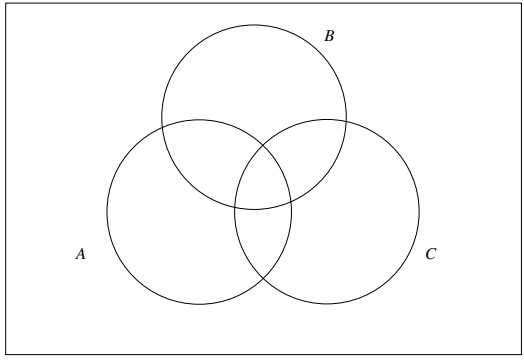

$$A \cup (A \cap B) = A$$



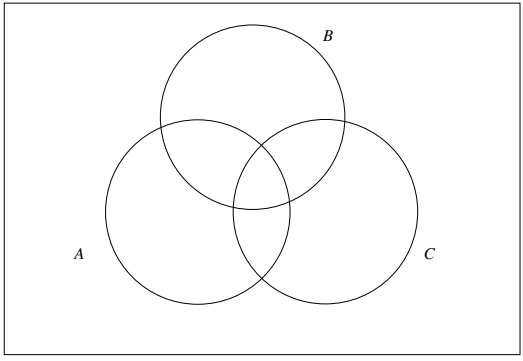



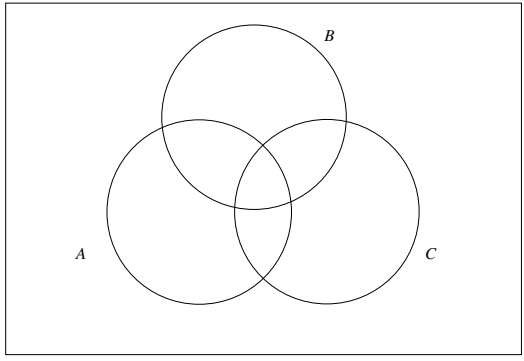

$$A \cup \overline{A} = \mathcal{U}$$



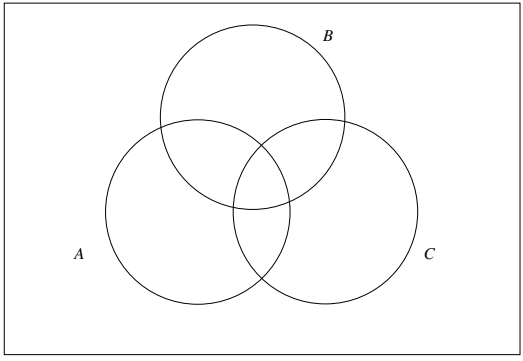



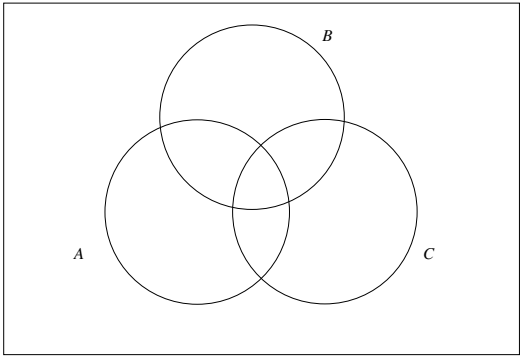

$$A \cup (B \cup C) = (A \cup B) \cup C$$



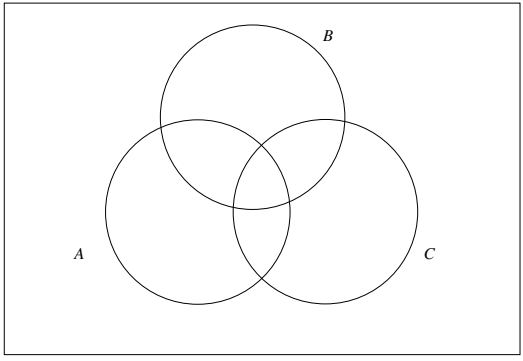



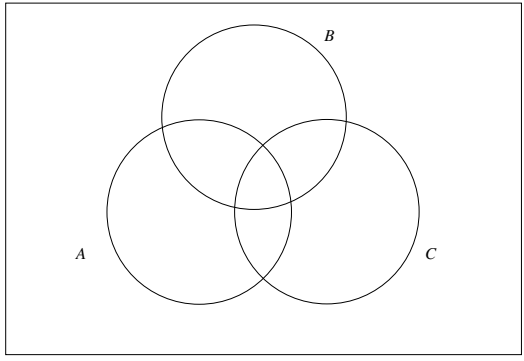

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$



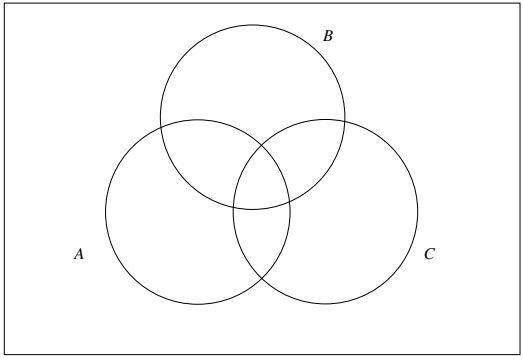



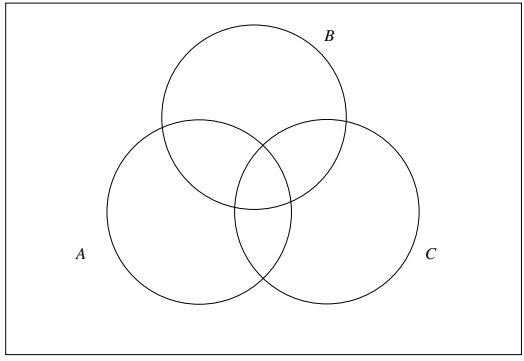

$$A \cap B = A - (A - B)$$



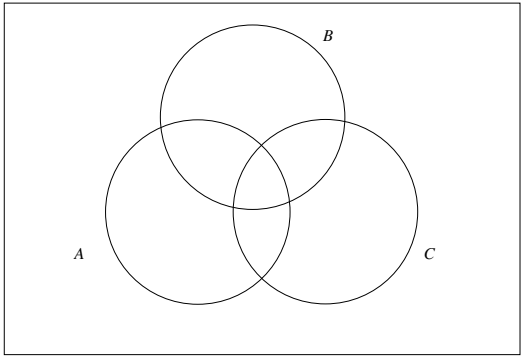



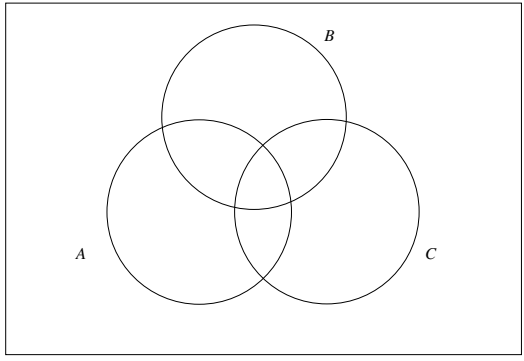

$$(A \cap C) - (C - B) = A \cap B \cap C$$




$$A \cup (A - B) = A$$




$$(A \cup (B - C)) \cap \overline{B} = A - B$$

**For next time:**

*Pg 37: 1.5.(14, 15, 16, 17, 24, 25)*
*Pg 43: 1.6.(5, 7, 9, 11, 13)*

*Highlighted exercises are programming exercises to be found in an accompanying notebook.*

*Read 1.(7 & 8)*

*Take quiz*