

Chapter 3, Case Studies:

- ▶ Linear-time sorting algorithms (last week Wednesday and Friday)
- ▶ Disjoint sets and array forests (**Today**)
- ▶ Priority queues and heaps (Wednesday and Friday)
- ▶ N -sets and bit vectors (Thursday lab)
- ▶ (Begin Graph unit in lab next week)

Today:

- ▶ Quiz solutions
- ▶ Problem statement
- ▶ Disjoint set ADT details
- ▶ The array forest abstraction and data structure
- ▶ Find and union strategies, with optimizations

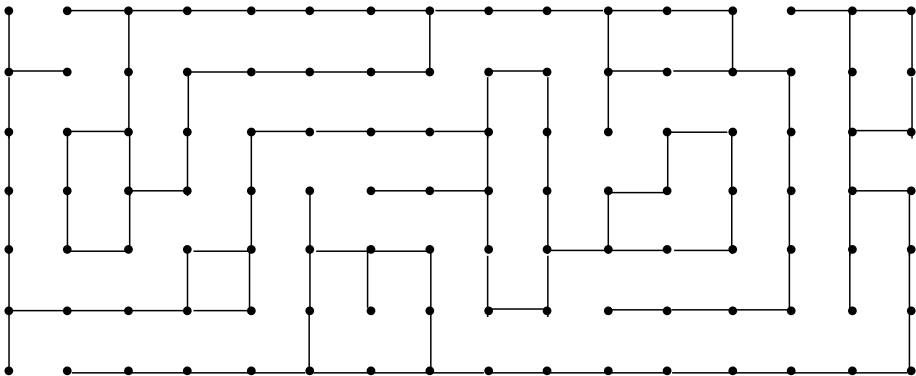
```
static Node arrayToList1(int[] array) {
    Node toReturn = new Node(array[0], null);
    for (int i = 1; i < array.length; i++) {
        Node current = toReturn;
        while (current.next() != null)
            current = current.next();
        current.setNext(new Node(array[i], null));
    }
    return toReturn;
}
```

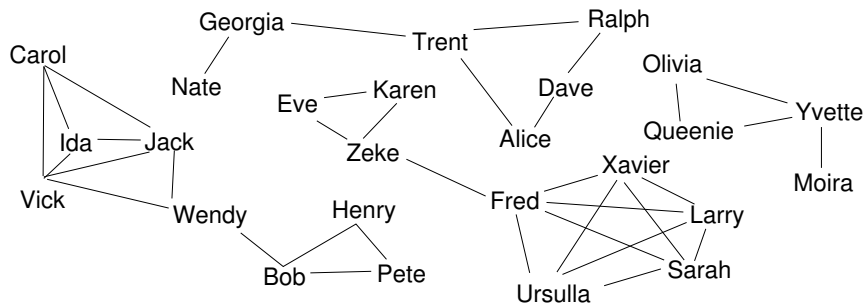
```
Node arrayToList2(int[] array) {
    Node toReturn = null;
    for (int i = array.length - 1; i >= 0; i--)
        toReturn = new Node(array[i], toReturn);
    return toReturn;
}
```

```
static int[] listToArray(Node head) {
    int size = 0;
    for (Node current = head; current != null; current = current.next())
        size++;
    int[] toReturn = new int[size];
    int i = 0;
    for (Node current = head; current != null; current = current.next())
        toReturn[i++] = current.datum();
    return toReturn;
}
```

Problem statement:

Suppose we have a collection of items connected by an unknown equivalence relation. Efficiently find the equivalence classes in this collection as information about the relation is discovered.

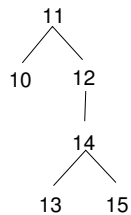
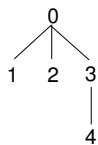
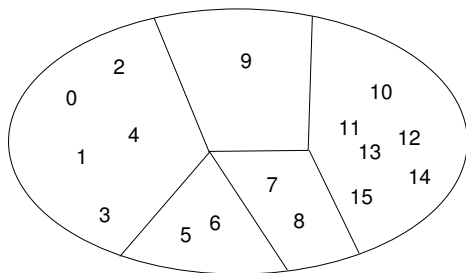


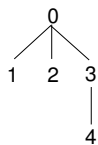
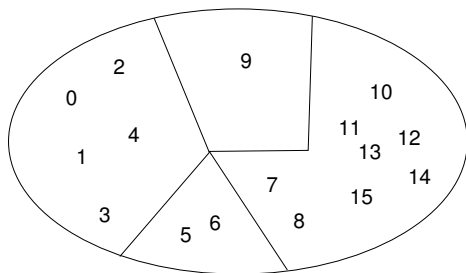


```
a = c
e = a + b
d = b
g = 1
f = d + c
h = e * g
```

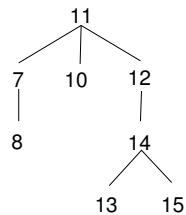
The *disjoint set* ADT:

- ▶ Main operations: union two sets, find a set for a given element, and test if two elements are in the same set.
- ▶ The universe is closed.
- ▶ We assume all elements can be indexed, $[0, N)$.
- ▶ A set in the partition is identified by a leader.





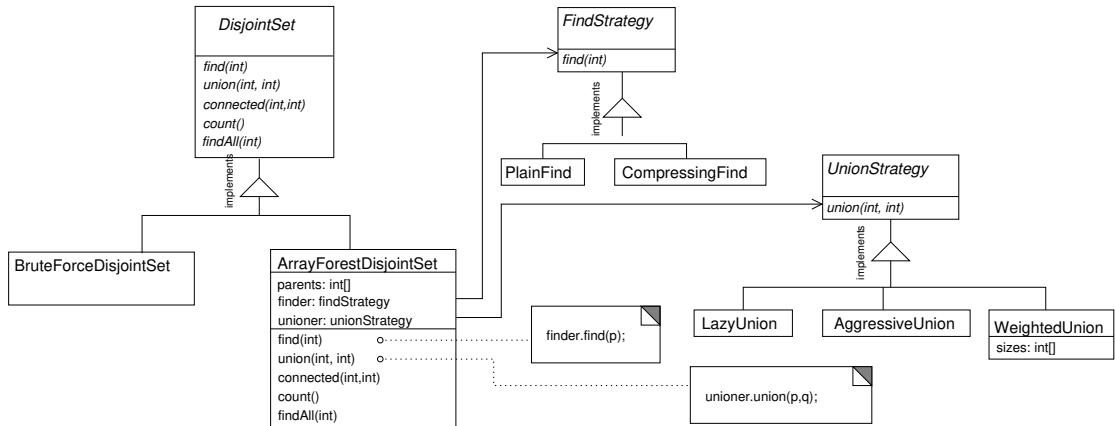
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Invariant (Class ArrayForestDisjointSet)

For all $i \in [0, n)$,

- (a) $leader(i) = leader(parents[i])$, that is, $parents[i]$ points to another element in the same set as i .
- (b) $leader(i) = parents[leader(i)]$, that is, leaders all point to themselves.
- (c) Following a finite number links implied by $parents$ will converge, that is, there is no circularity in the tree.



Union strategy	LazyUnion	AggressiveUnion	WeightedUnion	LazyUnion	WeightedUnion
Find strategy	PlainFind	PlainFind	PlainFind	CompressingFind	CompressingFind
Find heavy:	1.30E7 (5.68E6)	3.34E7 (8.40E3)	7.40E5 (1.80E4)	9.26E5 (2.38E4)	6.68E5 (9.34E3)
Even mix:	9.89E7 (1.22E7)	4.41E7 (9.93E3)	1.20E6 (1.97E4)	1.56E6 (2.12E4)	9.80E5 (9.96E3)
Union heavy:	1.62E8 (1.26E7)	4.39E7 (9.99E3)	1.40E6 (2.01E4)	1.71E6 (1.59E4)	1.04E6 (1.00E4)

Coming up:

*Do **linear sorting** project (Wed, Sept 24)*

*Due **Today:***

Finish reading Section 3.2 (disjoint sets and array forests)

Do Ex 2.(12 & 16) and 3.8 Take disjoint-sets quiz

*Due **Thurs, Sept 25:***

Read Section 3.4

Do Exercises 3.(26 & 27).

Take N-sets quiz

*Due **Fri, Sept 26:***

Read Section 3.3 (heaps and priority queues)

(no exercises)

Take heap/pq quiz