

## Chapter 4, Graphs:

- ▶ Concepts and implementation (Monday, Sept 29)
- ▶ Traversal (Wednesday, Oct 1 and in lab Thursday, Oct 2)
- ▶ Minimum spanning trees (Friday, Oct 3 and Monday, Oct 6)
- ▶ Single-source shortest paths (**Wednesday Oct 8, and Friday, Oct 10**)
- ▶ (Test 1 Wednesday, Oct 15)

### “Today” (Wednesday and Friday):

- ▶ (MST loose ends)
- ▶ The SSSP problem
- ▶ General concepts for SSSP algorithms
- ▶ The most unlucky graph for SSSP
- ▶ The Bellman-Ford algorithm plus analysis
- ▶ Dijkstra's algorithm plus analysis

## Minimum Spanning Tree Problem

Given a weighted, undirected graph, find the tree with least-total weight that connects all the vertices, if one exists.

- ▶ Both are defined for weighted graphs
- ▶ Both produce trees as a result
- ▶ Both minimize by weight
- ▶ For each we have two algorithms

Input is only a graph

Problem usually is described on an undirected graph

Goal is to minimize total weight

There is no clear winner between the algorithms

## Single-Source Shortest Paths Problem

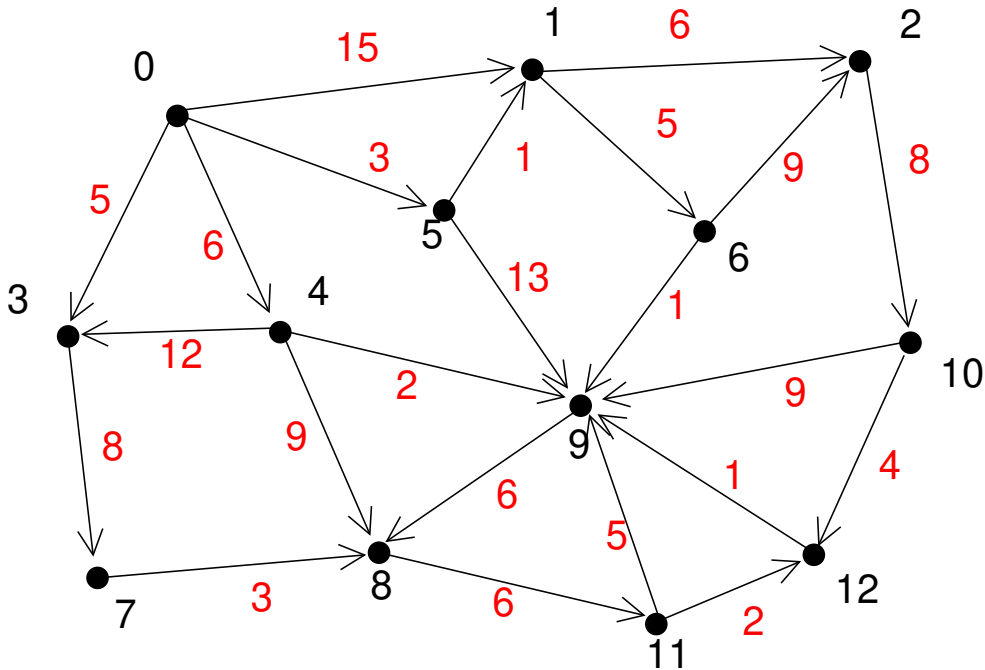
Given a weighted directed graph and a source vertex, find the tree comprising the shortest paths from that source to all other reachable vertices.

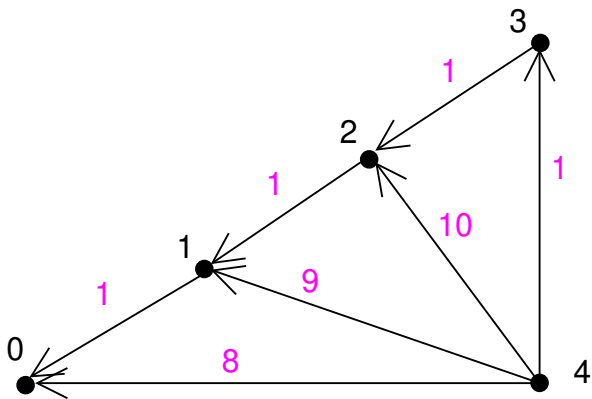
Input is a graph and a starting point

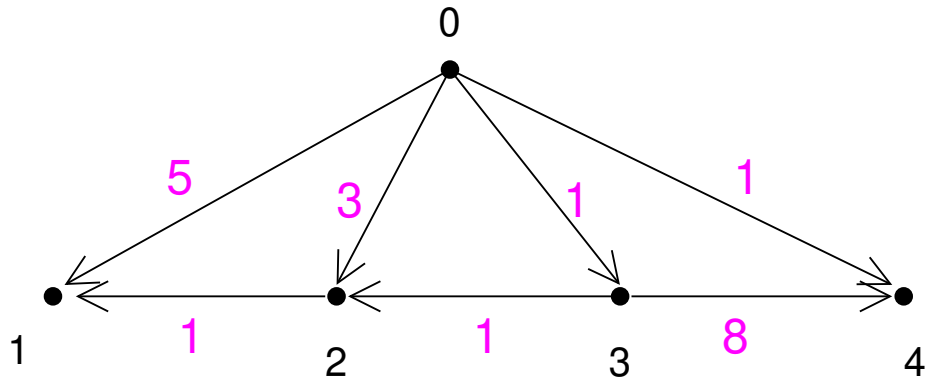
Problem usually is described on a directed graph

Goal is to minimize weight on each path

One algorithm is clearly more efficient







Let  $X$  be the set of vertices whose distance bounds are correct, that is,  $v \in X$  if  $\text{distances}[v]$  is the total weight of the shortest path from  $s$  to  $v$ . For a single-source shortest path algorithm to be correct, all vertices reachable from  $s$  are in set  $X$  at termination, and if *all* vertices are reachable, this implies  $X = V$ . Let  $Y$  be the set of vertices that have been removed from the priority queue. Our intent is that  $Y \subseteq X$ : all vertices have correct distance bound at the time they are removed from the priority queue, though at any point there may also be some correct ones still in the priority queue. We claim

### Invariant (Main loop of Dijkstra's algorithm)

*Let  $X$  and  $Y$  be as defined above.*

- (a)  $Y \subseteq X$ .
- (b) If  $v$  is the vertex in the priority queue with least distance bound, then  $v \in X$ .
- (c)  $|Y|$  is the number of iterations completed.

## Coming up:

*Do **MST** project (due Wednesday, Oct 8)*

*Do **SSSP** project (due Friday, Oct 17)*

*Due **Fri, Oct 10***

*Read Section 4.5*

*Do Exercises 4.(55–57)*

*Take SSSP quiz*