Nondeterminism

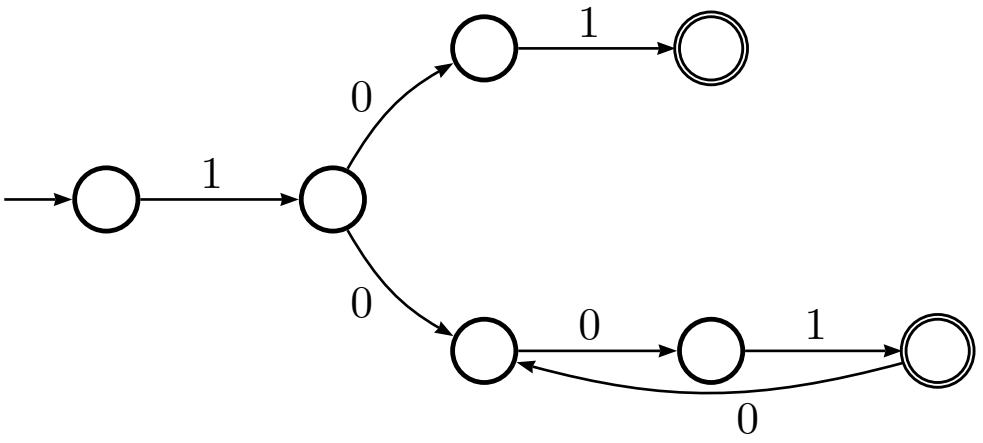
Recall that a *finite state machine* is also called a *deterministic finite accepter* and is defined as a tuple $(Q, \Sigma, \delta, q_0, F)$, where Q is a set of states, Σ is an input alphabet, $\delta : Q \times \Sigma \to Q$ is a state transition function, q_0 is the initial state, and F is the set of accept states.

nondeterministic finite accepter

A *nondeterministic finite accepter* (NFA) is like a DFA except that the state transition function δ is different. Now $\delta : Q \times \Sigma \to \mathscr{P}(Q)$. That is, δ maps to a set of states rather than a single state. Alternately, δ could be viewed as a relation from $(Q \times \Sigma)$ to Q. The idea is that for a state, symbol pair, there may be more than possible next state.

A NFA is said to *accept* a string $s_1 s_2 \ldots s_n$ if there exists a sequence of states $q_1 q_2 \ldots q_n$ such that for all i, $1 \leq i \leq n$, $q_i \in \delta(q_{i-1}, s_i)$ and $q_n \in F$. In other words, there only needs to be a possible set of transitions that could be made while reading the input that will result in an accept state. It doesn't matter if there also exists a set of transitions for the same input that does not lead to an accept state; we still consider the NFA to accept the string.

Suppose we want to define a language containing the strings "101", "1001", "1001001", "1001001001", etc. That is, strings beginning with 1 followed by one or more occurrences of the substring "001", with the exception that the initial 1 may be followed by "01" only. The following diagram shows a NFA that accepts this.



Non-determinism is difficult to talk about intuitively if we want to use the analogy that the automaton is like a real-world machine. Here are three ways to imagine it:

- When reaching a choice point in its operation, the automaton simply knows, oracularly, the right state to which it should transition in order to reach an accept state, if any.
- When the automaton reaches a choice point, the universe diverges into two (or more) parallel universes, each representing the universe in which the automaton has made one of the possible choices. Those universes themselves may diverge. If in any universe the automaton reaches an accept state, then that universe is the one where existence continues.
- Rather than splitting the universe, suppose that when a choice point is reached the automaton itself splits into two or more automata, and each goes its own way. Soon you'll have a bifurcating herd of automata; if any of the herd reaches an accept state, then the string is accepted.

Notice that the language we mentioned above is also accepted by the following DFA: