



$$C[24][2] = \min \begin{cases} \text{use 13-ducat coin, } 1 + C[11][2] \\ \text{don't use 13-ducat coin, } C[24][1] \end{cases}$$

Generalized:

$$C[i][j] = \min \begin{cases} 1 + C[i - D[j]][j] & \text{use the } j\text{th denomination,} \\ & \text{which value } D[j] \\ C[i][j - 1] & \text{skip the } j\text{th denomination} \end{cases}$$

Recursive case (how many of the  $j$ th denomination coins should we take when making change for amount  $i$ ?)

$$C[i][j] = \min_{0 \leq k < x} \{k + C[i - k \cdot D[j]][j - 1]\}$$

$x$  is the exclusive upper bound on how many  $j$ th denomination coins we *could* take, the smallest  $x$  such that  $x \cdot D[j] > i$ .

Base case:

$$C[i][0] = i$$

