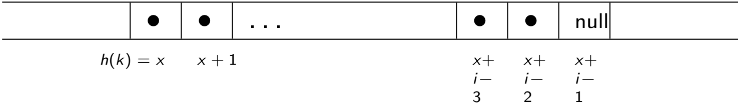
What is the probability that a search miss will require i probes?



How likely is it that position $h(k)$ is filled? $\frac{n}{m}$.

How likely is it that position $h(k) + 1$ is filled? Seems like $\frac{n-1}{m-1}$.

The probability that a search miss will require *at least* i probes is

$$\frac{n}{m} \cdot \frac{n-1}{m-1} \cdot \frac{n-2}{m-2} \cdots \frac{n-i+2}{m-i+2}$$

$$\leq \left(\frac{n}{m}\right)^{i-1}$$

$$= \alpha^{i-1}$$

Where does the inequality come from? Observe that

$$\begin{aligned}
n &< m \\
-m &< -n \\
mn - m &< mn - n \\
m(n-1) &< n(m-1) \\
\frac{n-1}{m-1} &< \frac{n}{m}
\end{aligned}$$

So, the expected number of probes is a weighted average of all possible number of probes:

$$\sum_{i=1}^{\infty} i \cdot P(\substack{\text{it takes} \\ i \text{ probes}}) = \sum_{i=1}^{\infty} i \cdot \left(P\begin{pmatrix}\text{it takes} \\ \text{at least} \\ i \text{ probes}\end{pmatrix} - P\begin{pmatrix}\text{it takes at} \\ \text{least } i-1 \\ \text{probes}\end{pmatrix}\right)$$

$$= \sum_{i=1}^{\infty} P\begin{pmatrix}\text{it takes} \\ \text{at least} \\ i \text{ probes}\end{pmatrix} \leq \sum_{i=1}^{\infty} \alpha^{i-1}$$

$$= \sum_{i=0}^{\infty} \alpha^{i} = 1 + \alpha + \alpha^2 + \alpha^3$$

$$= \frac{1}{1-\alpha}$$