

For Exercise 4.10.3, name the function `pow`. For Exercise 4.10.4, name the function `mul`.

4.10.6 Let the base- $b$  *floor logarithm* ( $\text{flog}$ ) of  $x$  be the greatest integer less than  $\log_b x$ . Define the base- $b$  *remainder logarithm* ( $\text{rlog}$ ) of  $x$  to be the difference between  $x$  and  $b$  raised to the floor log of  $x$ . For examples,

	<b>floor log</b>	<b>remainder log</b>
17 = 16 + 1		
= 2 <sup>4</sup> + 1	$\text{flog}_2 17 = 4$	$\text{rlog}_2 17 = 1$
30 = 27 + 3		
= 3 <sup>3</sup> + 3	$\text{flog}_3 30 = 3$	$\text{rlog}_3 30 = 3$
68 = 64 + 4		
= 4 <sup>3</sup> + 4	$\text{flog}_4 68 = 3$	$\text{rlog}_4 68 = 4$

**Lemma 1** For all  $a, b \in \mathbb{N}$ , there exists unique  $n, r \in \mathbb{W}$  such that  $a = b^n + r$  and  $0 \leq r < (b - 1) \cdot b^n$ .

**Lemma 2** For all  $b, n, r \in \mathbb{W}$ ,  $b^n + r = b^{n+1} + r - (b - 1) \cdot b^n$ .

Using these lemmas, write an ML function `frlog` that takes two integers ( $a$  and  $b$ ) and returns  $\text{flog}_b a$  and  $\text{rlog}_b a$  as a tuple.