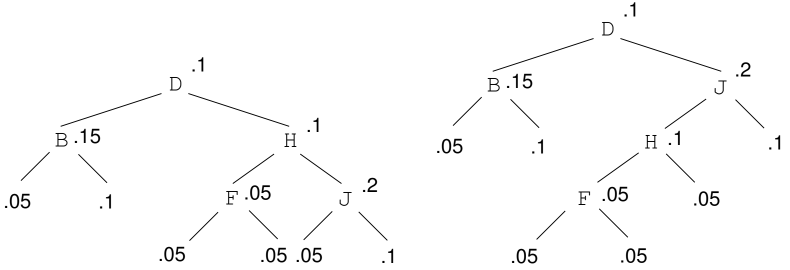
		0		1		2		3		4	
Keys (k_i)		B		D		F		H		J	
Key probabilities (p_i)		.15		.1		.05		.1		.2	
Miss probabilities (q_i)	.05		.1		.05		.05		.05		.1
	0		1		2		3		4		5

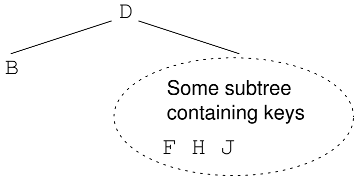


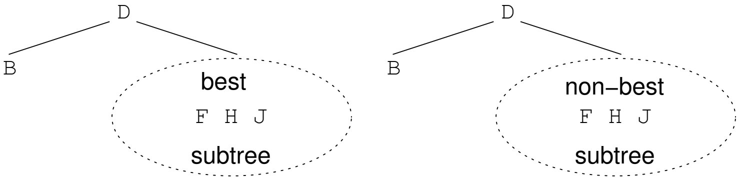
For the left tree:

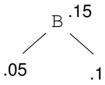
$$.1+2(.15+.1)+3(.05+.1+.05+.2)+4(.05+.05+.05+.1)=2.8$$

For the right tree:

$$.1{+}2(.15{+}.2){+}3{+}(.05{+}.1{+}.1{+}.1){+}5(.05{+}.05){+}5(.05{+}.05)=2.75$$

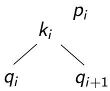

D
B
Some subtree
containing keys
F H J


D
B
best
F H J
subtree
D
B
non−best
F H J
subtree



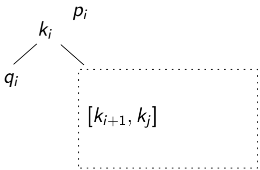
total probability: $.05 + .15 + .1 = .3$

total weighted depth: $2 \cdot .05 + .15 + 2 \cdot .1 = .45$



total probability: $q_i + p_i + q_{i+1}$

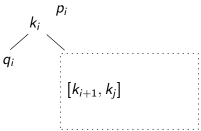
total weighted depth: $2 \cdot q_i + p_i + 2 \cdot q_{i+1}$



total probability: $q_i + p_i +$ total prob $[k_{i+1}, k_j]$

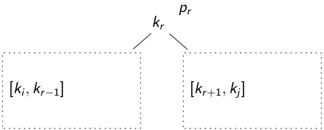
total weighted depth: $2 \cdot q_i + p_i + ???$

$$
\begin{aligned}
& \sum_{x=i}^{j} p_x \cdot depth_1(k_x) + \sum_{x=i}^{j+1} q_x \cdot depth_1(d_x) \\
= \ & 2 \cdot q_i + p_i + \sum_{x=i+1}^{j} p_x \cdot depth_1(k_x) + \sum_{x=i+1}^{j+1} q_x \cdot depth_1(d_x) \\
= \ & 2 \cdot q_i + p_i + \sum_{x=i+1}^{j} p_x \cdot (depth_0(k_x) + 1) + \sum_{x=i+1}^{j+1} q_x \cdot (depth_0(d_x) + 1) \\
= \ & 2 \cdot q_i + p_i + \sum_{x=i+1}^{j} p_x + \sum_{x=i+1}^{j+1} q_x + \sum_{x=i+1}^{j} p_x \cdot depth_0(k_x) + \sum_{x=i+1}^{j+1} q_x \cdot depth_0(d_x) \\
= \ & q_i + q_i + p_i + \sum_{x=i+1}^{j} p_x + \sum_{x=i+1}^{j+1} q_x + (\text{total weighted depth of subtree}) \\
= \ & q_i + (\text{total prob of tree}) + (\text{total weighted depth of subtree})
\end{aligned}
$$



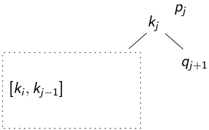
total probability: $q_i + p_i + T[i+1][j]$

total weighted depth: $q_i + T[i][j] + C[i+1][j]$



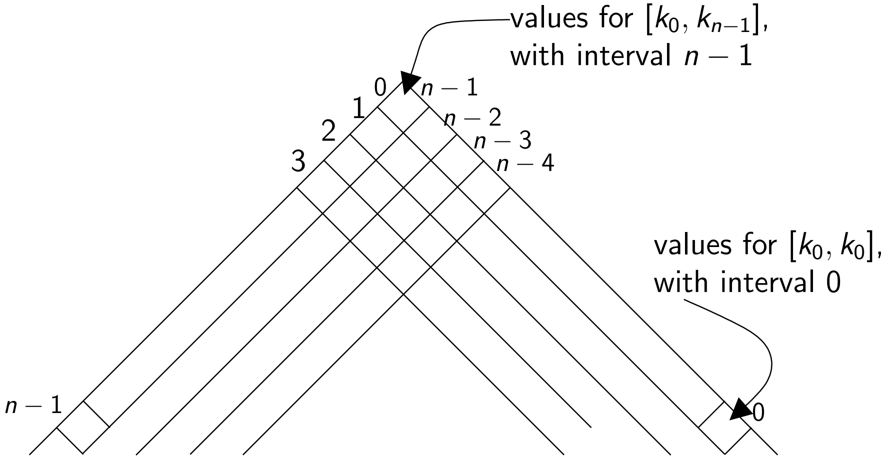
total probability: $T[i][r-1] + p_r + T[r+1][j]$

total weighted depth: $C[i][r-1] + T[i][j] + C[r+1][j]$



total probability: $T[i][j-1] + p_j + q_{j+1}$

total weighted depth: $C[i][j-1] + T[i][j] + q_{j+1}$


values for $[k_0, k_{n-1}]$,
with interval $n-1$
0
1
2
3
$n-1$
$n-2$
$n-3$
$n-4$
$n-1$
values for $[k_0, k_0]$,
with interval 0
0

For each diagonal from bottom up

For each (i, j) in that diagonal

Determine $T[i][j]$, $C[i][j]$, and the root of the best tree

Initialize the cells $(0, 0)$ through $(n-1, n-1)$

For each interval size from 1 to $n-1$

- For each (i, j) in that interval ($j = i + \text{interval}$)
    - Find $T[i][j]$
    - Consider each key k_r,
        - keeping track of the root of the best tree seen so far
        - and associated total weighted depth
        - Special case for k_i ($r = i$)
            - Compute total weighted depth, assume it's best so far
        - For each $k_r \in [k_{i+1}, \ldots k_{j-1}]$ (each $r \in [i+1, \ldots j-1)$
            - Compute total weighted depth, compare with best so far
        - Special case for k_j ($r = j$)
            - Compute total weighted depth, compare with best so far
    - Enter table entries for (i, j)

Return tree rooted at cell $(0, n-1)$ in node tree