

Valid argument

If it is Monday, then it is raining
It is Monday.
Therefore it is raining.

$p \rightarrow q$

p

$\therefore q$

p	q	$p \rightarrow q$	q
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	F

Annotations:
- Above the first column: p (premise)
- Above the second column: q (premise)
- Above the third column: $p \rightarrow q$ (premise)
- Above the fourth column: q (conclusion)
- An arrow labeled "critical row" points to the second row (where p is true and q is false).

Invalid argument

If it is raining, then there are clouds
There are clouds.
Therefore it is raining.

$p \rightarrow q$

q

$\therefore p$

p	q	$p \rightarrow q$	p	
T	T	T	T	\leftarrow critical row
T	F	F	T	
F	T	T	F	\leftarrow critical row
F	F	T	F	

Alternate definition of validity

Valid argument

p	q	$p \rightarrow q$	$(p \wedge (p \rightarrow q)) \rightarrow q$
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	T

Invalid argument

p	q	$p \rightarrow q$	$(q \wedge (p \rightarrow q)) \rightarrow p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

Modus tollens

If it is spring, then the daffodils bloom.
The daffodils aren't blooming.
Therefore it is not spring.

p	q	$p \rightarrow q$	$\sim q$	$\sim p$
T	T	T	F	
T	F	F	T	
F	T	T	F	
F	F	T	T	T

WINNIE-THE-POOH

tily. Through copse and spinney marched Bear; down open slopes of gorse and heather, over rocky beds of streams, up steep banks of sandstone into the heather again; and so at last, tired and hungry, to the Hundred Acre Wood. For it was in the Hundred Acre Wood that Owl lived.

"And if anyone knows anything about anything," said Bear to himself, "it's Owl who knows something about something," he said, "or my name's not Winnie-the-Pooh," he said. "Which it is," he added. "So there you are."

Owl lived at The Chestnuts, an old-world residence of great charm, which was grander than anybody else's, or seemed so to Bear, because it had both a knocker *and* a bell-pull. Underneath the knocker there was a notice which said:

PLES RING IF AN RNSER IS REQIRD.

Underneath the bell-pull there was a notice which said:

PLEZ CNOKE IF AN RNSR IS NOT REQID.

These notices had been written by Christopher Robin, who was the only one in the forest who could spell; for Owl, wise though he was in many ways, able to read and write and spell his own name WOL, yet somehow went all to pieces over delicate words like MEASLES and BUTTERED TOAST.

the words 'EAT ME' were beautifully marked in currants. 'Well, I'll eat it,' said Alice, 'and if it makes me grow larger, I can reach the key; and if it makes me grow smaller, I can creep under the door; so either way I'll get into the garden, and I don't care which happens!'

She ate a little bit, and said anxiously to herself, 'Which way? Which way?', holding her hand on the top of her head to feel which way it was growing, and she was quite surprised to find that she remained the same size: to be sure, this generally happens when one eats cake, but Alice had got so much into the way of expecting nothing but out-of-the-way things to happen, that it seemed quite dull and stupid for life to go on in the common way.

So she set to work, and very soon finished off the cake.

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Lewis Carroll, 1865.
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3.9.1

(a) $t \rightarrow u$

(b) $p \vee \sim q$

(c) $p \rightarrow (u \rightarrow r)$

(d) q

(e) $\therefore t \rightarrow r$

3.9.2

(a) $p \rightarrow t$

(b) $\sim (q \rightarrow t) \rightarrow w$

(c) $p \vee q$

(d) $\sim w$

(e) $\therefore t$

3.9.8

(a) w

(b) $q \rightarrow r$

(c) $t \rightarrow s$

(d) $u \rightarrow s$

(e) $(\sim t \wedge \sim u) \rightarrow \sim w$

(f) $(s \vee y) \rightarrow (p \rightarrow q)$

(g) $\sim (p \rightarrow r) \vee x$

(h) $\therefore x$

3.9.9

(a) $p \rightarrow q$

(b) x

(c) $\sim (p \vee w) \rightarrow r$

(d) $q \rightarrow u$

(e) $x \rightarrow t$

(f) $w \rightarrow u$

(g) $r \vee s$

(h) $r \rightarrow F$

(i) $\therefore t \wedge s \wedge u$

3.9.10

(a) $u \rightarrow \sim p$

(b) $(\sim p \vee q) \rightarrow (r \rightarrow s)$

(c) $u \wedge \sim w$

(d) $t \rightarrow s$

(e) $(\sim t \wedge \sim r) \rightarrow w$

(f) $\therefore s$