$\{(1, 2), (2, 3), (5, 2), (1, 5), (2, 5), (1, 3)\}$

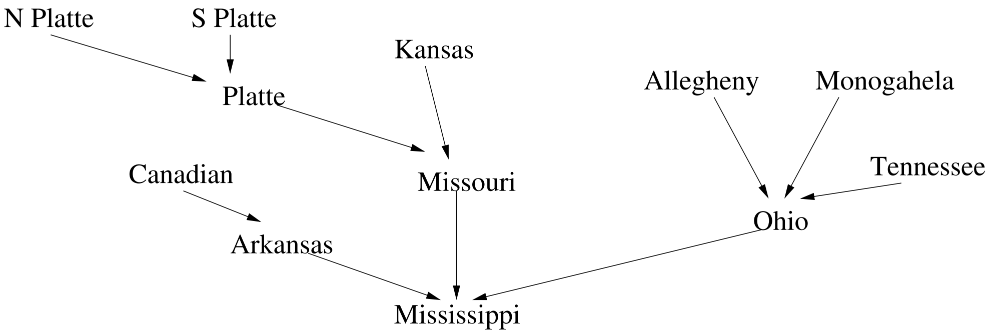
$\{(1, 2), (2, 3), (5, 2), (1, 5), (2, 5), (1, 3)\}$

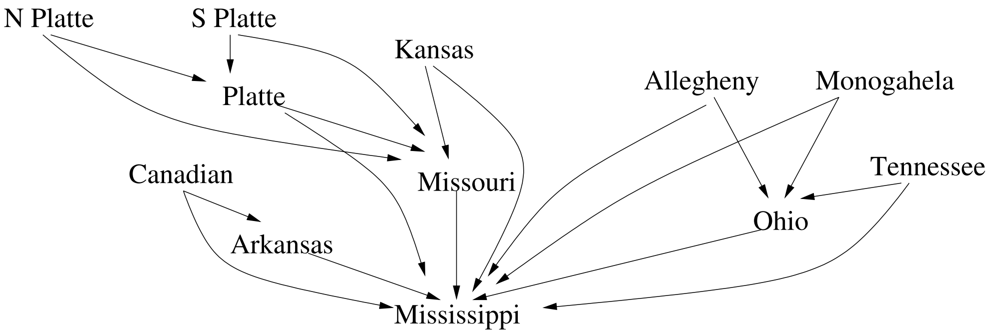
$\{(1, 2), (2, 3), (5, 2), (1, 5), (2, 5), (1, 3)\}$

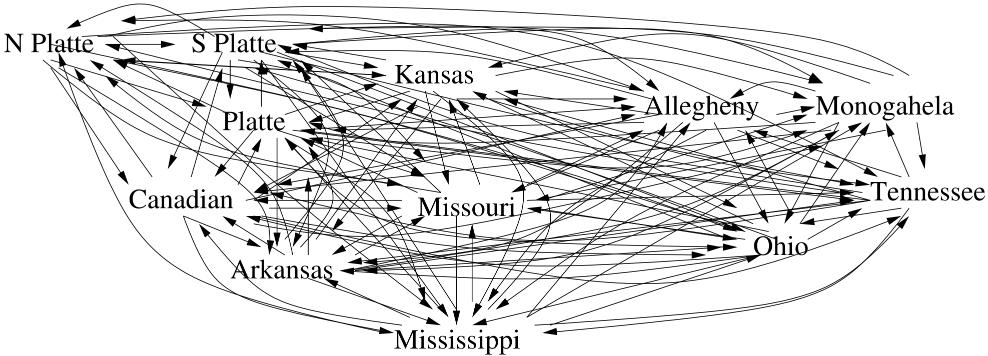
$\{(1, 2), (2, 3), (5, 2), (1, 5), (2, 5), (1, 3)\}$

Domain	First relation	Second relation
Rivers	*flows into* The Platte flows into the Missouri, and the Missouri flows into the Mississippi.	*is tributary to* The Platte is a tributary to the Missouri; both the Platte and the Missouri are tributaries to the Mississippi.
People	*is parent of* Bill is Jane's parent; Jane is Leroy's parent	*is ancestor of* Bill is Jane's ancestor; Leroy has both Jane and Bill as ancestors.

Domain	**First relation**	**Second relation**
Animals	*eats* Rabbit eats clover; coyote eats rabbit.	*derives nutrients from* Coyote derives nutrients from rabbit; rabbit derives nutrients from clover; both coyote and rabbit ultimately derive nutrients from clover.
$\mathbb{Z}$	*is one less than* 2 is one less than 3; 3 is one less than 4	$<$ $2 < 3$; $3 < 4$; $2 < 4$.


N Platte
S Platte
Kansas
Allegheny
Monogahela
Platte
Tennessee
Canadian
Missouri
Ohio
Arkansas
Mississippi


N Platte
S Platte
Kansas
Platte
Allegheny
Monogahela
Canadian
Missouri
Tennessee
Ohio
Arkansas
Mississippi


N Platte
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Kansas
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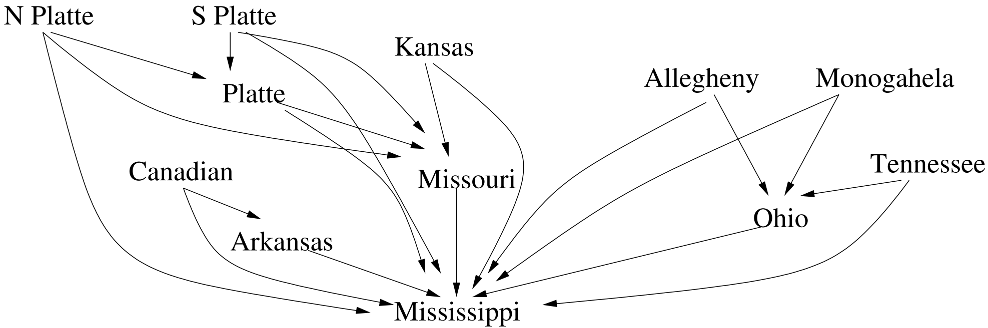
Mississippi


N Platte
S Platte
Kansas
Platte
Allegheny
Monogahela
Canadian
Missouri
Tennessee
Ohio
Arkansas
Mississippi

Theorem 5.12 *The transitive closure of a relation R is unique.*

Proof. *Suppose S and T are relations fulfilling the requirements for being transitive closures of R. By items 1 and 2, S is transitive and $R \subseteq S$, so by item 3, $T \subseteq S$. By items 1 and 2, T is transitive and $R \subseteq T$, so by item 3, $S \subseteq T$. Therefore $S = T$ by the definition of set equality.* $\square$

Theorem 5.13 *If R is a relation on a set A, then*

$$R^{\infty} = \bigcup_{i=1}^{\infty} R^i = \{(x, y) \mid \exists\ i \in \mathbb{N} \text{ such that } (x, y) \in R^i\}$$

is the transitive closure of R.

Proof. *Suppose R is a relation on a set A.*
Suppose $a, b, c \in A$, $(a, b), (b, c) \in R^{\infty}$. By the definition of R^{∞}, there exist $i, j \in \mathbb{N}$ such that $(a, b) \in R^i$ and $(b, c) \in R^j$. By the definition of relation composition and Exercise 5.7.4, $(a, c) \in R^j \circ R^i = R^{i+j}$. $R^{i+j} \subseteq R^{\infty}$ by the definition of R^{∞}. By the definition of subset, $(a, c) \in R^{\infty}$. Hence, R^{∞} is transitive by definition.
Suppose $a, b \in A$ and $(a, b) \in R$. By the definition of R^{∞} (taking $i = 1$), $(a, b) \in R^{\infty}$, and so $R \subseteq R^{\infty}$, by definition of subset.
Suppose S is a transitive relation on A and $R \subseteq S$. Further suppose $(a, b) \in R^{\infty}$. Then, by definition of R^{∞}, there exists $i \in \mathbb{N}$ such that $(a, b) \in R^i$. By Lemma 5.14, $(a, b) \in S$. Hence $R^{\infty} \subseteq S$ by definition of subset.
Therefore, R^{∞} is the transitive closure of R. $\square$