

0-1 Knapsack.

Given a capacity c and the value and weight of n items in arrays V and W , find a subset of the n items whose total weight is less than or equal to the capacity and whose total value is maximal.

V	20	15	90	100
W	1	2	4	5
	0	1	2	3

$c = 7$

set	weight	value	
{2, 3}	9	190	exceeds capacity
{1, 3}	7	115	not optimal
{0, 1, 2}	7	125	optimal

$$B[i][j] = \begin{cases} 0 & \text{if } j = 0 \text{ and } W[0] > i \\ V[0] & \text{if } j = 0 \text{ and } W[0] \leq i \\ B[i][j - 1] & \text{if } W[j] > i \\ \max \left\{ \begin{array}{l} V[j] + B[i - W[j]][j - 1], \\ B[i][j - 1] \end{array} \right\} & \text{otherwise} \end{cases}$$

3	20/S	20/S	20/S	90/S	110/S	120/T	125/S
2	20/S	20/S	20/S	90/T	110/T	110/T	125/T
1	20/S	20/S	35/T	35/T	35/T	35/T	35/T
0	20/T	20/T	20/T	20/T	20/T	20/T	20/T

Longest common subsequence.

Given two sequences, find the longest subsequence that they have in common.

D	A	T	A	S	T	R	U	C	T	U	R	E	S
A	L	G	O	R	I	T	M	S					

A	A	A	A	A	B	not	A	A	A	A	A	B
A	B	A	A	A	A		A	B	A	A	A	A

A	A	A	A	A	B	A	A	A	A	not	A	A	A	A	B	A	A	A	A
A	B	A	A	A	A	A		A	B	A	A	A	A	A					

$$L[i][j] = \begin{cases} 0 & \text{if } i = 0 \text{ or } j = 0 \\ 1 + L[i - 1][j - 1] & \text{if } i \neq 0 \text{ and } j \neq 0 \text{ and } a[i] = b[j] \\ \max\{L[i][j - 1], \\ \quad L[i - 1][j]\} & \text{otherwise} \end{cases}$$

T	0	0	1	2	2	3	4	4	4	4	5
E	0	0	1	2	2	3	4	4	4	4	4
H	0	0	1	2	2	3	3	3	3	3	3
C	0	0	1	2	2	2	2	2	2	2	2
T	0	0	1	1	1	1	1	1	1	1	2
A	0	0	1	1	1	1	1	1	1	1	1
R	0	0	0	0	0	0	0	0	0	0	0
	B	L	A	C	K	H	E	I	G	H	T

$$M[i][j] = \begin{cases} 0 & \text{if } i = j \\ \min_{i \leq k < j} \{ M[i][k] + \\ & D[i] \times D[k+1] \times D[j+1] + \\ & M[k+1][j] \} & \text{otherwise} \end{cases}$$