

## 0-1 Knapsack.

Given a capacity  $c$  and the value and weight of  $n$  items in arrays  $V$  and  $W$ , find a subset of the  $n$  items whose total weight is less than or equal to the capacity and whose total value is maximal.

$V$	20	15	90	100
$W$	1	2	4	5
	0	1	2	3

$$c = 7$$

set	weight	value	
$\{2, 3\}$	9	190	<i>exceeds capacity</i>
$\{1, 3\}$	7	115	<i>not optimal</i>
$\{0, 1, 2\}$	7	125	<i>optimal</i>

## Longest common subsequence.

*Given two sequences, find the longest subsequence that they have in common.*

D	A	T	A	S	T	R	U	C	T	U	R	E	S
A	L	G	O	R	I	T	M	S					

A	A	A	A	A	B		A	A	A	A	A	B
A	B	A	A	A	A	not	A	B	A	A	A	A

A	A	A	A	A	B	A	A	A	A		A	A	A	A	A	B	A	A	A	A
A	B	A	A	A	A					not	A	A	A	A	A	A				

## Matrix multiplication.

*Given  $n + 1$  dimensions of  $n$  matrices to be multiplied, find the optimal order in which to multiply the matrices, that is, find the parenthesization of the matrices that will minimize the number of scalar multiplications.*

Assume the following matrices and dimensions:  $A, 3 \times 5$ ;  $B, 5 \times 10$ ;  $C, 10 \times 2$ ,  $D, 2 \times 3$ ;  $E, 3 \times 4$ .

$$(A \times B) \times (C \times (D \times E)) \quad 3 \cdot 5 \cdot 10 + 2 \cdot 3 \cdot 4 + 10 \cdot 2 \cdot 4 + 3 \cdot 10 \cdot 4 = 374$$

$$(A \times (B \times C)) \times (D \times E) \quad 5 \cdot 10 \cdot 2 + 2 \cdot 3 \cdot 4 + 3 \cdot 5 \cdot 2 + 3 \cdot 2 \cdot 4 = 178$$

$$A \times (B \times (C \times (D \times E))) \quad 2 \cdot 3 \cdot 4 + 10 \cdot 2 \cdot 4 + 5 \cdot 10 \cdot 4 + 3 \cdot 5 \cdot 4 = 364$$

<i>Problem</i>	<i>Thing to find</i>	<i>Optimization</i>	<i>Constraint</i>
Coin-changing	A set of coins.	Minimize the number of coins.	The coins' values sum to the given amount.
Knapsack	A set of objects	Maximize the sum of the objects' values.	The sum of the objects' weights doesn't exceed the given capacity.
Longest common subsequence	A subsequence in each of two given sequences.	Maximize the length of the subsequences.	The subsequences have the same content.
Matrix multiplication	A way to parenthesize the the matrices being multiplied.	Minimize the number of scalar multiplications required.	The parenthesization is complete and mathematically coherent.
Optimal BST	A BST for a given set of keys	Minimize the expected length of a search.	The tree satisfies the criteria for a BST.