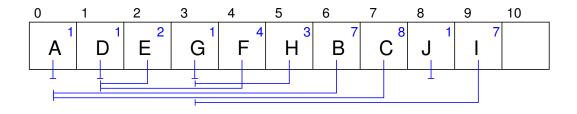
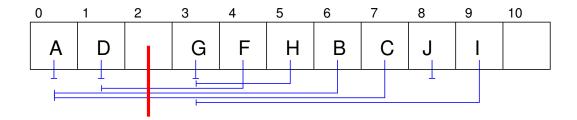
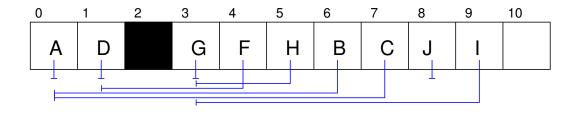
## Invariant (Class OpenAddressingHashMap)

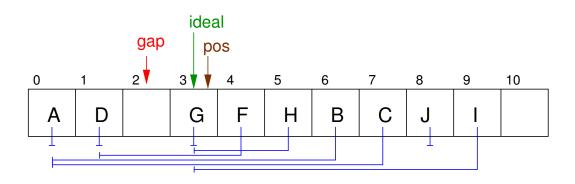
- 1. The table it not full; there exists  $i \in [0, m)$  such that table [i] = null.
- 2. There are no breaks in the chain for any key in the table; for all  $i \in [0, m)$  such that table[i] contains key k,
  - if  $h(k) \le i$ , then for all  $j \in [h(k), i]$ , table $[j] \ne null$ ;
  - if i < h(k), then for all  $j \in [0, i] \cup [h(k), m)$ .

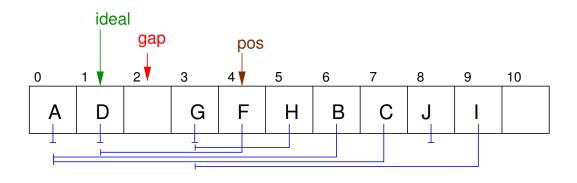


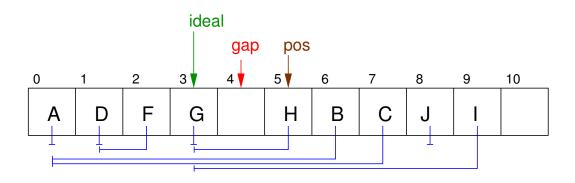


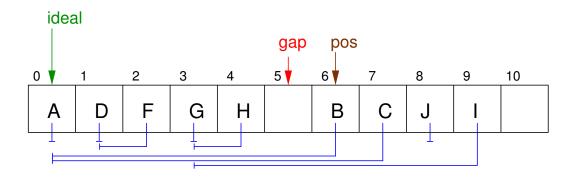


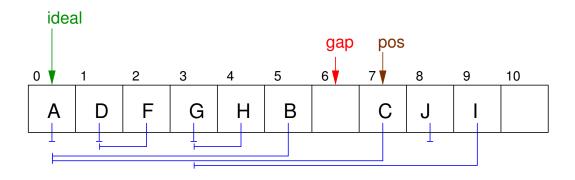


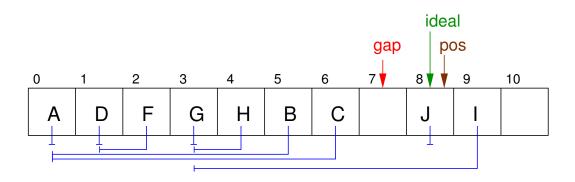


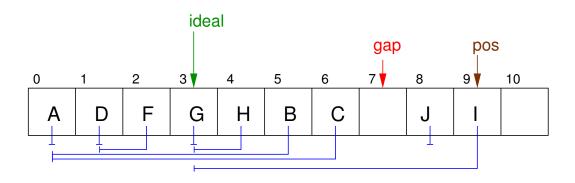


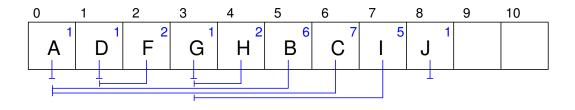


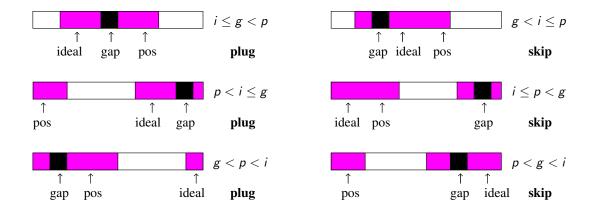












## Invariant (Loop of optimized remove in linear probing.)

For all positions  $k \in (i,j)$ , gap is the only position, if any, between its ideal place (h(keys[k])) and its actual place (k).