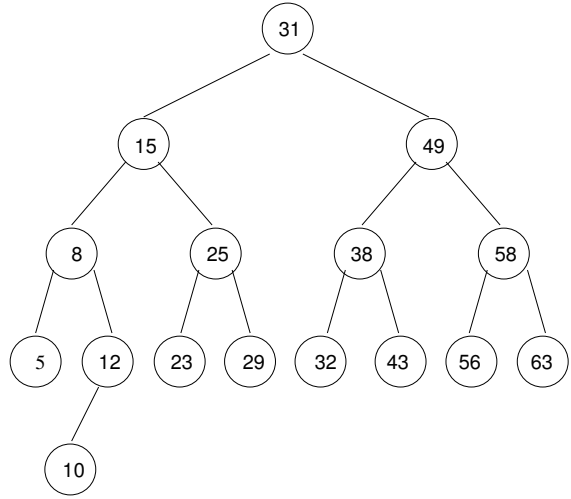
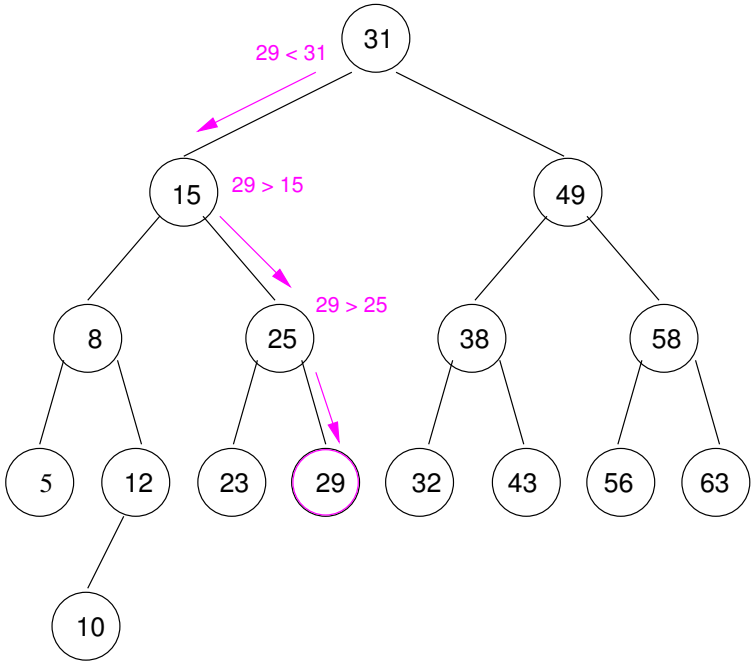
- empty, or
- a node augmented with a key k together with two children, designated *left* and *right*, such that
  - either *left* is empty or it is a binary search tree such that all of the keys in that tree are less than or equal to k, and
  - either *right* is empty or it is a binary search tree such that all of the keys in that tree are greater than or equal to k.

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29 > 15
29 > 25
31
15
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8
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58
5
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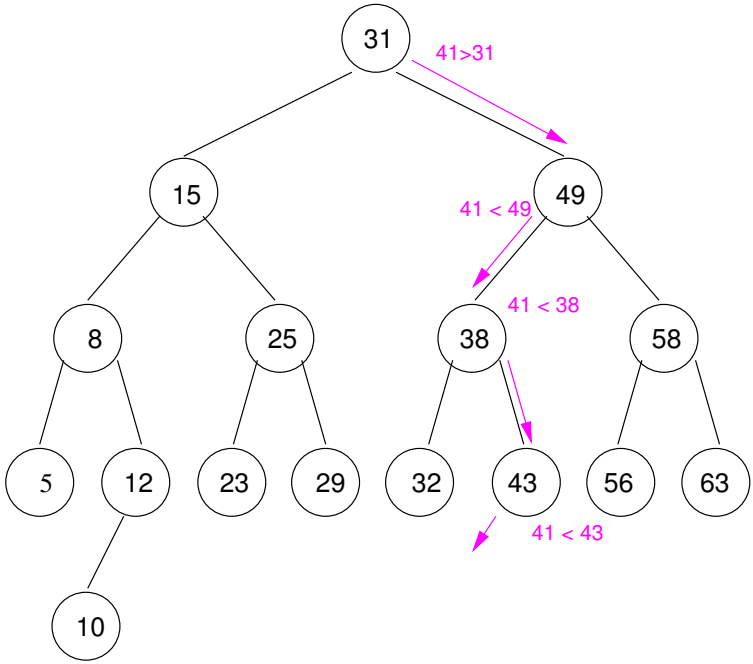
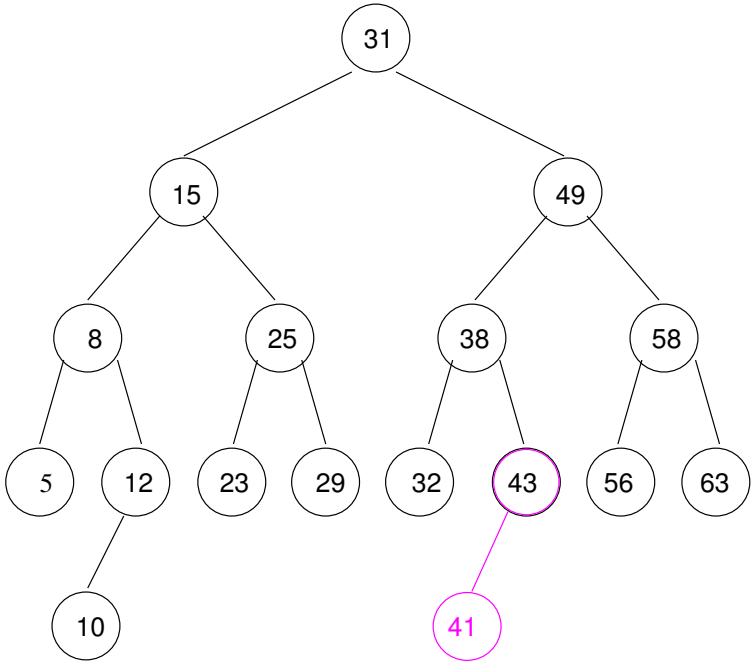
29
32
43
56
63
10


31
41>31
15
49
41 < 49
8
25
38
41 < 38
58
5
12
23
29
32
43
56
63
41 < 43
10


31
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49
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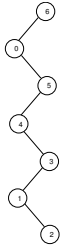
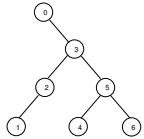
32
43
56
63
10
41

6, 0, 5, 1, 4, 2, 3



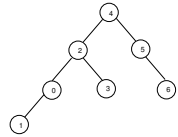
height 6
total depth 21
ANI 5

0, 3, 5, 2, 6, 1, 4



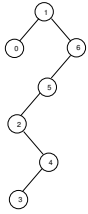
height 3
total depth 14
ANI 3

4, 2, 5, 3, 0, 1, 6



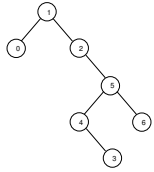
height 3
total depth 11
ANI 2.57

1, 6, 5, 2, 4, 3, 0



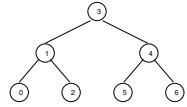
height 5
total depth 16
ANI 3.29

1, 2, 5, 4, 3, 0, 6

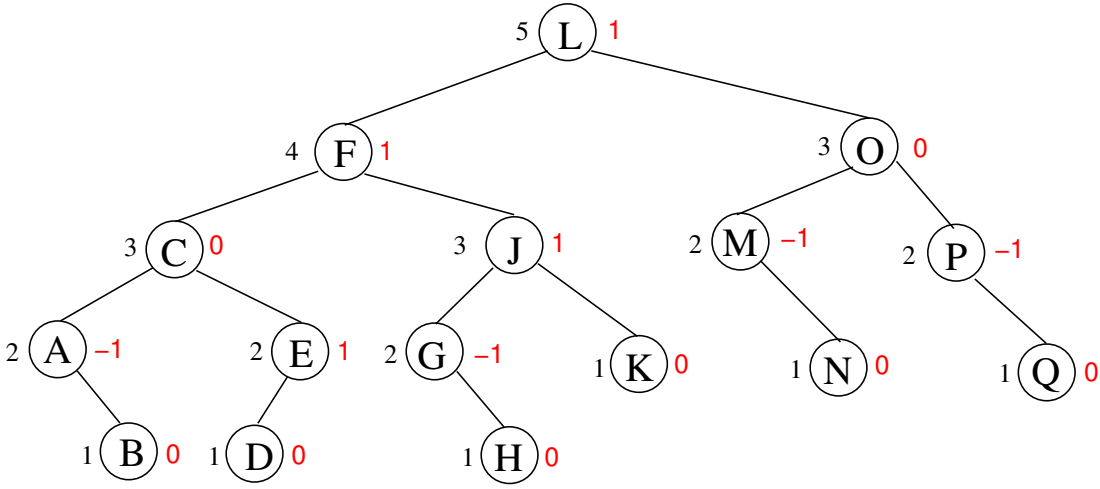


height 4
total depth 14
ANI 3

3, 1, 4, 0, 2, 5, 6



height 2
total depth 10
ANI 2.43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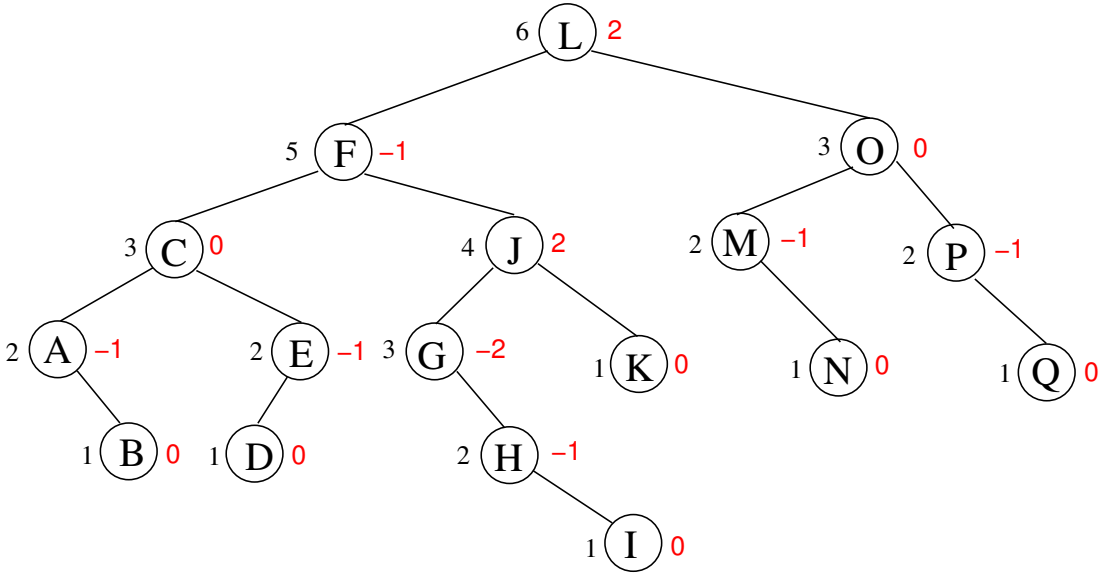
3 O 0
3 C 0
3 J 1
2 M −1
2 P −1
2 A −1
2 E 1
2 G −1
1 K 0
1 N 0
1 Q 0
1 B 0
1 D 0
1 H 0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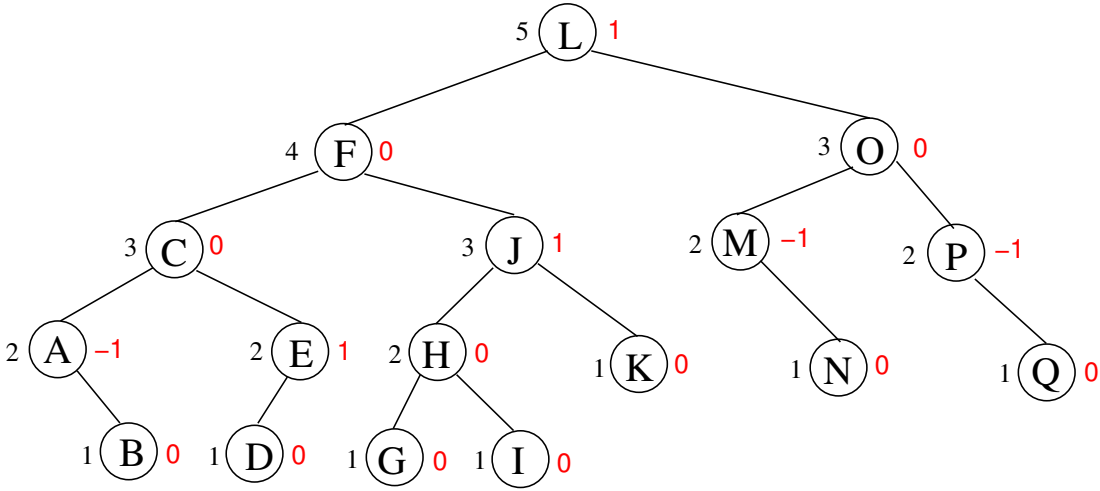
3 O 0
3 C 0
4 J 2
2 M −1
2 P −1
2 A −1
2 E −1
3 G −2
1 K 0
1 N 0
1 Q 0
1 B 0
1 D 0
2 H −1
1 I 0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4 F 0
3 O 0
3 C 0
3 J 1
2 M −1
2 P −1
2 A −1
2 E 1
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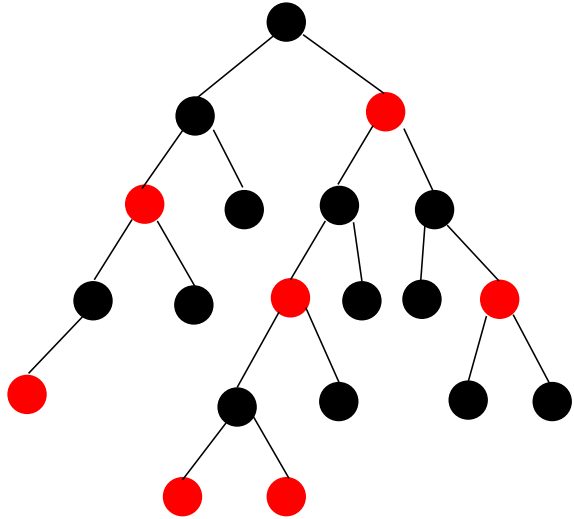
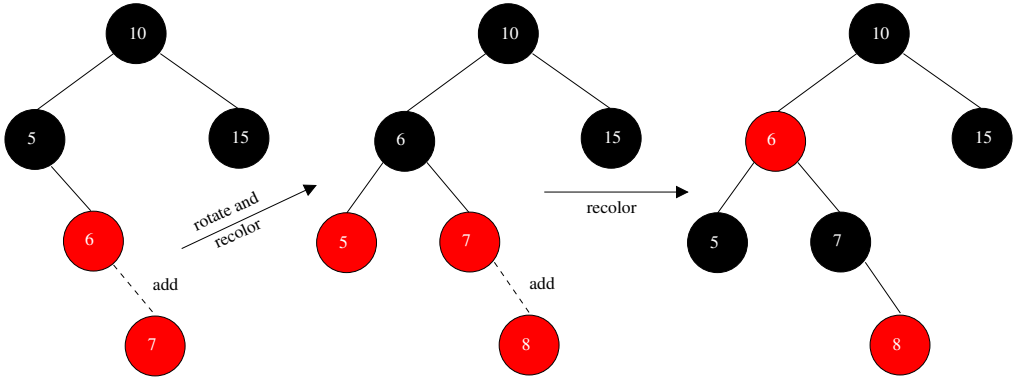
1 K 0
1 N 0
1 Q 0
1 B 0
1 D 0
1 G 0
1 I 0

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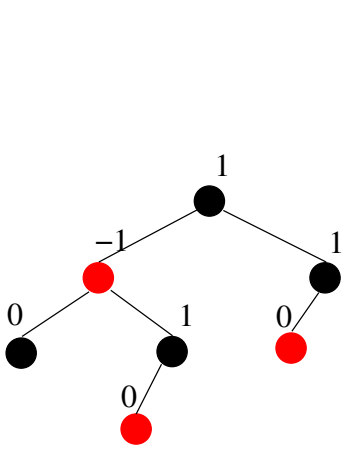
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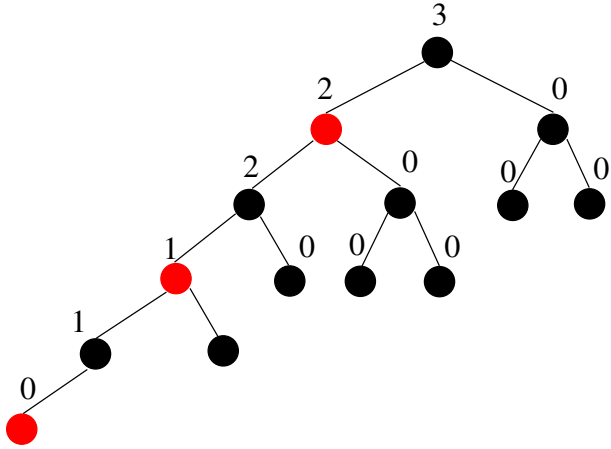
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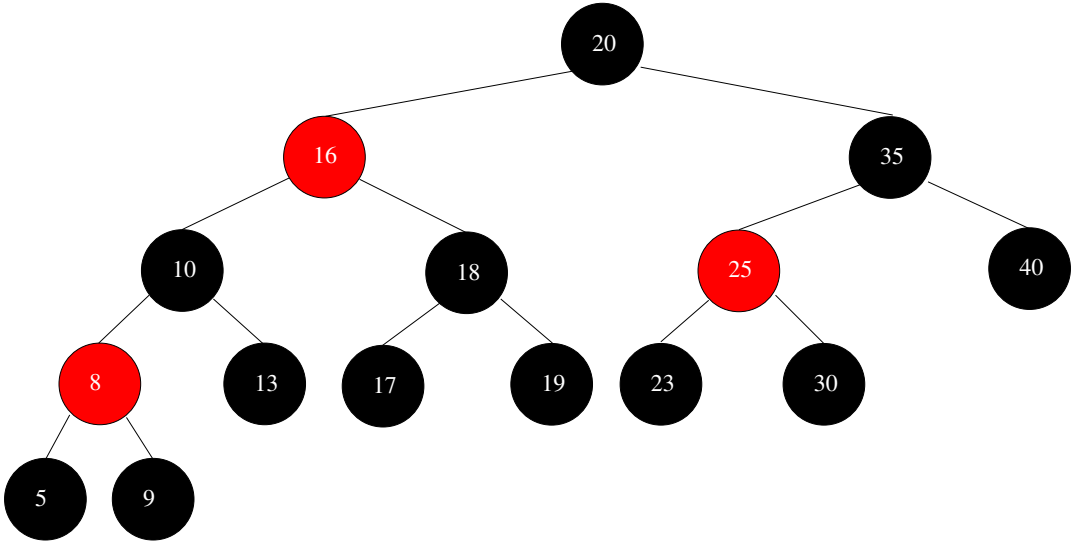
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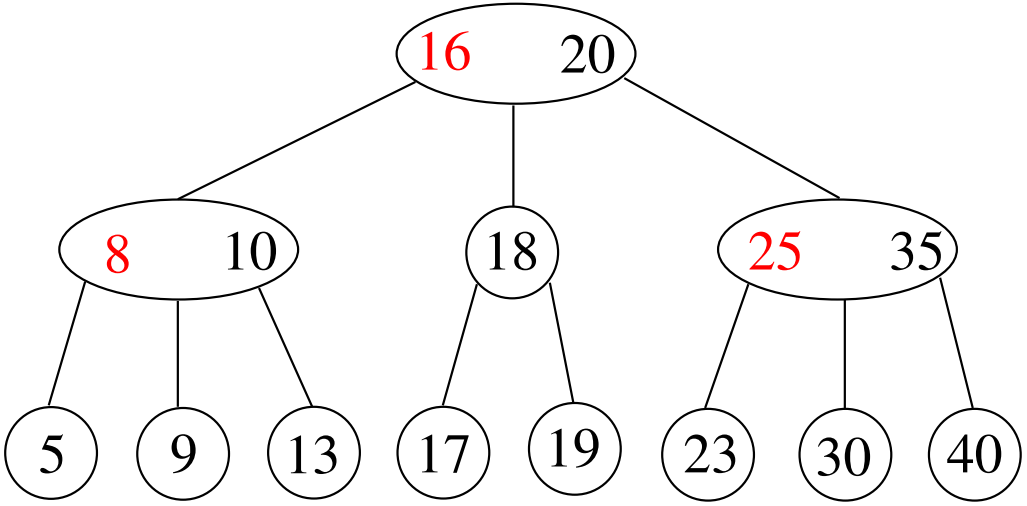
16 20
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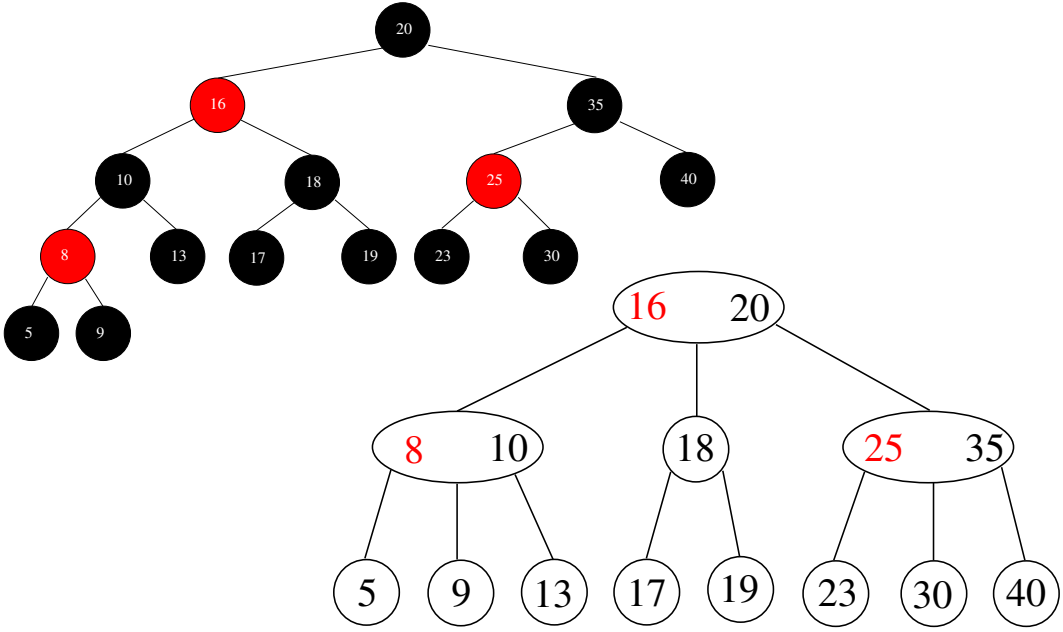
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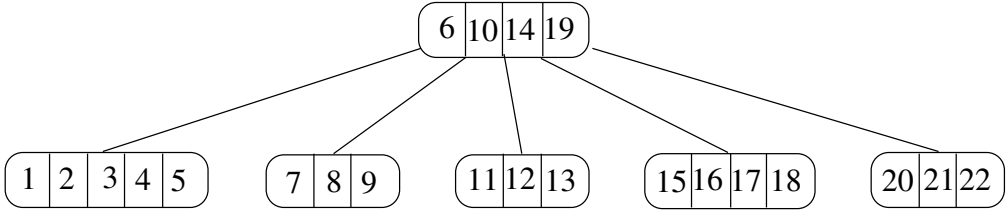
6 10 14 19
1 2 3 4 5
7 8 9
11 12 13
15 16 17 18
20 21 22