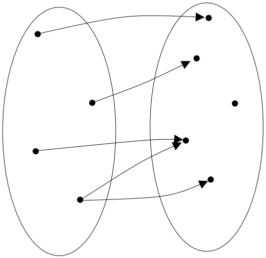
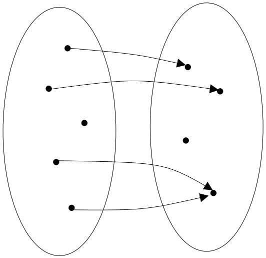
A relation f from X to Y is a function (written $f : X \to Y$) if $\forall\, x \in X$, (1) $\exists\, y \in Y \mid (x, y) \in f$, and (2) $\forall\, y_1, y_2 \in Y$, $(x, y_1), (x, y_2) \in f \to y_1 = y_2$.



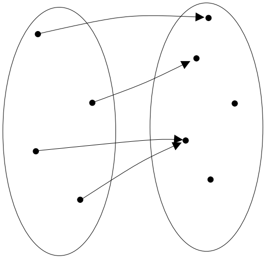
Not a function.

(There's a domain element that is related to two things.)



Not a function.

(There's a domain element that is not related to anything.)

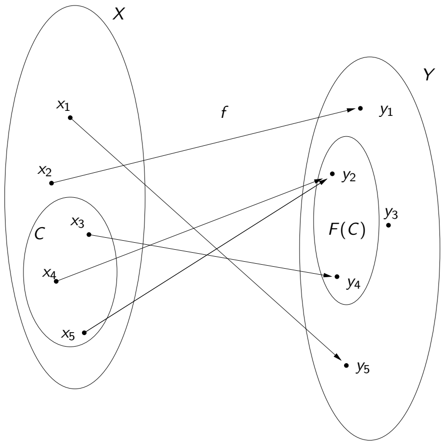


A function.

(It's OK that two domain elements are related to the same thing and one codomain element has nothing related to it.)

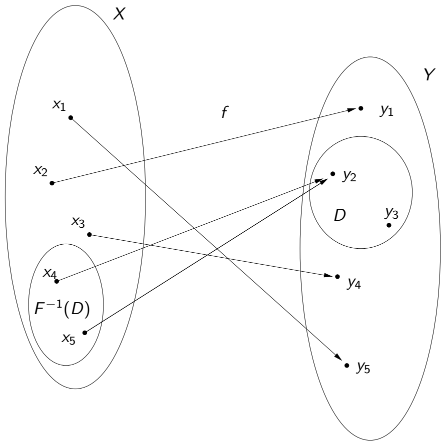
Image

$$F(A) = \{y \in Y \mid \exists\, x \in A \text{ such that } f(x) = y\}$$



Inverse image

$$F^{-1}(B) = \{x \in X \mid f(x) \in B\}$$



Lemma 7.2. If $f : X \to Y$, then $F(\emptyset) = \emptyset$.

Lemma 7.3. If $f : X \to Y$, $A \subseteq X$, and $A \neq \emptyset$, then $F(A) \neq \emptyset$.

Lemma 7.4. If $f : X \to Y$, then $F^{-1}(\emptyset) = \emptyset$.

We might expect the following, but *it's not true*:

Lemma XXXX. If $f : X \to Y$, $A \subseteq Y$, and $A \neq \emptyset$, then $F^{-1}(A) \neq \emptyset$.