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$$4|0 \qquad \qquad 0 + 1 = 1 = 5^0$$

$$4|4 \qquad \qquad 4 + 1 = 5 = 5^1$$

$$4|24 \qquad \qquad 24 + 1 = 25 = 5^2$$

$$4|124 \qquad \qquad 124 + 1 = 125 = 5^3$$

$$4|624 \qquad \qquad 624 + 1 = 625 = 5^4$$

$$\sum_{i=1}^1 i = 1 = 1 = \frac{1 \cdot 2}{2}$$

$$\sum_{i=1}^2 i = 1 + 2 = 3 = \frac{2 \cdot 3}{2}$$

$$\sum_{i=1}^3 i = 1 + 2 + 3 = 6 = \frac{3 \cdot 4}{2}$$

$$\sum_{i=1}^4 i = 1 + 2 + 3 + 4 = 10 = \frac{4 \cdot 5}{2}$$

$$\sum_{i=1}^5 i = 1 + 2 + 3 + 4 + 5 = 15 = \frac{5 \cdot 6}{2}$$

$$\text{Ex 6.6.1. } \forall n \in \mathbb{N}, \overline{\bigcup_{i=1}^n A_i} = \bigcap_{i=1}^n \overline{A_i}$$

Proof. By induction on n .

Base case. Suppose $n = 1$. Then

$$\overline{\bigcup_{i=1}^1 A_i} = \overline{A_1} = \bigcap_{i=1}^1 \overline{A_i}$$

Inductive case. Suppose $\overline{\bigcup_{i=1}^n A_i} = \bigcap_{i=1}^n \overline{A_i}$ for some $n \geq 1$. Then

$$\overline{\bigcup_{i=1}^{n+1} A_i} = \overline{A_{n+1} \cup \bigcup_{i=1}^n A_i} \quad \text{by definition of iterated union}$$

$$= \overline{A_{n+1}} \cap \overline{\bigcup_{i=1}^n A_i} \quad \text{by Ex 4.3.13 (DeMorgan's law of sets)}$$

$$= \overline{A_{n+1}} \cap \bigcap_{i=1}^n \overline{A_i} \quad \text{by the inductive hypothesis}$$

$$= \bigcap_{i=1}^{n+1} \overline{A_i} \quad \text{by the definition of iterated intersection}$$

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