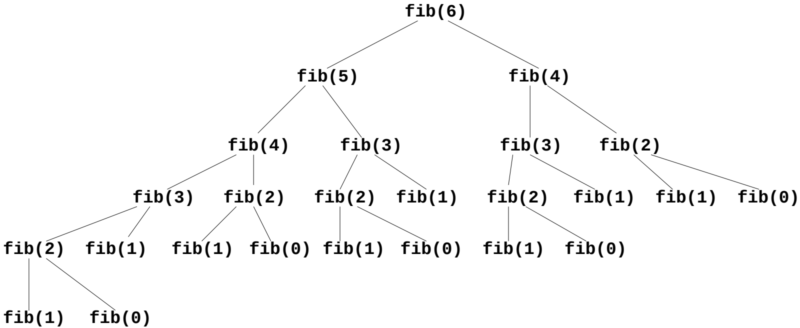
fib(6)
fib(5)
fib(4)
fib(4)
fib(3)
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fib(3)
fib(2)
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fib(1)
fib(2)
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fib(1)
fib(0)
fib(1)
fib(0)

$$C[i][j] = \begin{cases} 0 & \text{if } i = 0 \\ i & \text{if } j = 0 \\ \min_{0 \le k < \frac{i}{D[j]}} \{k + C[i - k \cdot D[j]][j-1]\} & \text{otherwise} \end{cases}$$

0-1 Knapsack.

Given a capacity c and the value and weight of n items in arrays V and W, find a subset of the n items whose total weight is less than or equal to the capacity and whose total value is maximal.

	0	1	2	3
V	20	15	90	100
W	1	2	4	5

$c = 7$

set	weight	value	
$\{2, 3\}$	9	190	*exceeds capacity*
$\{1, 3\}$	7	115	*not optimal*
$\{0, 1, 2\}$	7	125	*optimal*

Longest common subsequance.

Given two sequences, find the longest subsequence that they have in common.

D A T A S T R U C T U R E S
A L G O R I T M S

A A A A A B   not   A A A A A B
A B A A A A   A B A A A A

A A A A A B A A A A   not   A A A A A B A A A A
A B A A A A   A B A A A A

Matrix multiplication.

> *Given $n+1$ dimensions of of n matrices to be multiplied, find the optimal order in which to multiply the matrices, that is, find the parenthesization of the matrices that will minimize the number of scalar multiplications.*

Assume the following matrices and dimensions: $A, 3 \times 5$; $B, 5 \times 10$; $C, 10 \times 2$, $D, 2 \times 3$; $E, 3 \times 4$.

$$(A \times B) \times (C \times (D \times E)) \qquad 3 \cdot 5 \cdot 10 + 2 \cdot 3 \cdot 4 + 10 \cdot 2 \cdot 4 + 3 \cdot 10 \cdot 4 = 374$$

$$(A \times (B \times C)) \times (D \times E) \qquad 5 \cdot 10 \cdot 2 + 2 \cdot 3 \cdot 4 + 3 \cdot 5 \cdot 2 + 3 \cdot 2 \cdot 4 = 178$$

$$A \times (B \times (C \times (D \times E))) \qquad 2 \cdot 3 \cdot 4 + 10 \cdot 2 \cdot 4 + 5 \cdot 10 \cdot 4 + 3 \cdot 5 \cdot 4 = 364$$

Problem	*Thing to find*	*Optimization*	*Constraint*
Coin-changing	A set of coins.	Minimize the number of coins.	The coins' values sum to the given amount.
Knapsack	A set of objects	Maximize the sum of the objects' values.	The sum of the objects' weights doesn't exceed the given capacity.
Longest common subsequence	A subsequence in each of two given sequences.	Maximize the length of the subsequences.	The subsequences have the same content.
Matrix multiplication	A way to parenthesize the the matrices being multiplied.	Minimize the number of scalar multiplications required.	The parenthesization is complete and mathematically coherent.
Optimal BST	A BST for a given set of keys	Minimize the expected length of a search.	The tree satisfies the criteria for a BST.