

$$((q \wedge (p \wedge (p \vee q))) \vee (q \wedge \sim p)) \wedge \sim q$$

$$\equiv ((q \wedge p) \vee (q \wedge \sim p)) \wedge \sim q \quad \text{Absorption}$$

$$\equiv (q \wedge (p \vee \sim p)) \wedge \sim q \quad \text{Distributivity}$$

$$\equiv (q \wedge T) \wedge \sim q \quad \text{Negation}$$

$$\equiv q \wedge \sim q \quad \text{Identity}$$

$$\equiv F \quad \text{Negation}$$

**WRONG!**

$$((q \wedge (p \wedge (p \vee q))) \vee (q \wedge \sim p)) \wedge \sim q$$

$$\equiv ((q \wedge p) \vee (q \wedge \sim p)) \wedge \sim q \quad \text{Absorption}$$

$$\equiv (q \wedge p) \vee ((q \wedge \sim p) \wedge \sim q) \quad \text{Associativity}$$

# Valid argument

If it is Monday, then it is raining  
It is Monday.  
Therefore it is raining.

$p \rightarrow q$

$p$

$\therefore q$

$p$	$q$	$p \rightarrow q$	$q$
$T$	$T$	$T$	$T$
$T$	$F$	$F$	$F$
$F$	$T$	$T$	$T$
$F$	$F$	$T$	$F$

Annotations:  
- Above the first column:  $p$  (premise)  
- Above the second column:  $q$  (premise)  
- Above the third column:  $p \rightarrow q$  (premise)  
- Above the fourth column:  $q$  (conclusion)  
- An arrow labeled "critical row" points to the second row (where  $p$  is true and  $q$  is false).

# Invalid argument

If it is raining, then there are clouds  
There are clouds.  
Therefore it is raining.

$p \rightarrow q$

$q$

$\therefore p$

$p$	$q$	$p \rightarrow q$	$p$	
$T$	$T$	$T$	$T$	$\leftarrow$ critical row
$T$	$F$	$F$	$T$	
$F$	$T$	$T$	$F$	$\leftarrow$ critical row
$F$	$F$	$T$	$F$	

## Alternate definition of validity

Valid argument

$p$	$q$	$p \rightarrow q$	$(p \wedge (p \rightarrow q)) \rightarrow q$
$T$	$T$	$T$	$T$
$T$	$F$	$F$	$T$
$F$	$T$	$T$	$T$
$F$	$F$	$T$	$T$

Invalid argument

$p$	$q$	$p \rightarrow q$	$(q \wedge (p \rightarrow q)) \rightarrow p$
$T$	$T$	$T$	$T$
$T$	$F$	$F$	$T$
$F$	$T$	$T$	$F$
$F$	$F$	$T$	$T$

# Modus tollens

If it is spring, then the daffodils bloom.  
The daffodils aren't blooming.  
Therefore it is not spring.

$p$	$q$	$p \rightarrow q$	$\sim q$	$\sim p$
$T$	$T$	$T$	$F$	
$T$	$F$	$F$	$T$	
$F$	$T$	$T$	$F$	
$F$	$F$	$T$	$T$	$T$

### Modus Ponens

$$p \rightarrow q$$

$$p$$

$$\therefore q$$

### Modus Tollens

$$p \rightarrow q$$

$$\sim q$$

$$\therefore \sim p$$

### Generalization

$$p$$

$$\therefore p \vee q$$

### Specialization

$$p \wedge q$$

$$\therefore p$$

### Elimination

$$p \vee q$$

$$\sim p$$

$$\therefore q$$

### Transitivity

$$p \rightarrow q$$

$$q \rightarrow r$$

$$\therefore p \rightarrow r$$

### Division into cases

$$p \vee q$$

$$p \rightarrow r$$

$$q \rightarrow r$$

$$\therefore r$$

### Contradiction

$$p \rightarrow F$$

$$\therefore \sim p$$

# Proof by contradiction

$$p \rightarrow F$$
$$\therefore \sim p$$

$p$	$p \rightarrow F$	$\sim p$
$T$	$F$	$F$
$F$	$T$	$T$

$\leftarrow$  *critical row*



### 3.9.1

(a)  $t \rightarrow u$

(b)  $p \vee \sim q$

(c)  $p \rightarrow (u \rightarrow r)$

(d)  $q$

(e)  $\therefore t \rightarrow r$

### 3.9.2

(a)  $p \rightarrow t$

(b)  $\sim (q \rightarrow t) \rightarrow w$

(c)  $p \vee q$

(d)  $\sim w$

(e)  $\therefore t$

### 3.9.8

(a)  $w$

(b)  $q \rightarrow r$

(c)  $t \rightarrow s$

(d)  $u \rightarrow s$

(e)  $(\sim t \wedge \sim u) \rightarrow \sim w$

(f)  $(s \vee y) \rightarrow (p \rightarrow q)$

(g)  $\sim (p \rightarrow r) \vee x$

(h)  $\therefore x$

### 3.9.9

(a)  $p \rightarrow q$

(b)  $x$

(c)  $\sim (p \vee w) \rightarrow r$

(d)  $q \rightarrow u$

(e)  $x \rightarrow t$

(f)  $w \rightarrow u$

(g)  $r \vee s$

(h)  $r \rightarrow F$

(i)  $\therefore t \wedge s \wedge u$

### 3.9.10

(a)  $u \rightarrow \sim p$

(b)  $(\sim p \vee q) \rightarrow (r \rightarrow s)$

(c)  $u \wedge \sim w$

(d)  $t \rightarrow s$

(e)  $(\sim t \wedge \sim r) \rightarrow w$

(f)  $\therefore s$