$$((q \wedge (p \wedge (p \vee q))) \vee (q \wedge \sim p)) \wedge \sim q$$

$$\equiv ((q \wedge p) \vee (q \wedge \sim p)) \wedge \sim q \quad \text{Absorption}$$

$$\equiv (q \wedge (p \vee \sim p)) \wedge \sim q \quad \text{Distributivity}$$

$$\equiv (q \wedge T) \wedge \sim q \quad \text{Negation}$$

$$\equiv q \wedge \sim q \quad \text{Identity}$$

$$\equiv F \quad \text{Negation}$$

WRONG!

$$((q \wedge (p \wedge (p \vee q))) \vee (q \wedge \sim p)) \wedge \sim q$$

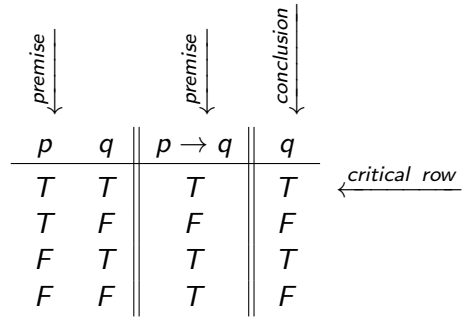
$$\equiv \quad ((q \wedge p) \vee (q \wedge \sim p)) \wedge \sim q \qquad \text{Absorption}$$

$$\equiv \quad (q \wedge p) \vee ((q \wedge \sim p) \wedge \sim q) \qquad \text{Associativity}$$

Valid argument

If it is Monday, then it is raining
It is Monday.
Therefore it is raining.

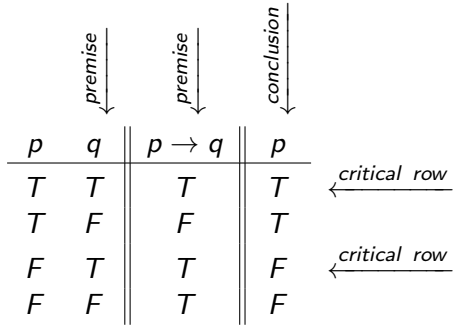
$p \rightarrow q$
p
$\therefore q$



Invalid argument

If it is raining, then there are clouds
There are clouds.
Therefore it is raining.

$p \rightarrow q$
q
$\therefore p$



Alternate definition of validity

Valid argument

p	q	$p \to q$	$(p \wedge (p \to q)) \to q$
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	T

Invalid argument

p	q	$p \to q$	$(q \wedge (p \to q)) \to p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

Modus tollens

If it is spring, then the daffodils bloom.
The daffodils aren't blooming.
Therefore it is not spring.

p	q	$p \to q$	$\sim q$	$\sim p$
T	T	T	F	
T	F	F	T	
F	T	T	F	
F	F	T	T	T

Modus Ponens

$p \to q$

p

$\therefore q$

Modus Tollens

$p \to q$

$\sim q$

$\therefore \sim p$

Generalization

p

$\therefore p \vee q$

Specialization

$p \wedge q$

$\therefore p$

Elimination

$p \vee q$

$\sim p$

$\therefore q$

Transitivity

$p \to q$

$q \to r$

$\therefore p \to r$

Division into cases

$p \vee q$

$p \to r$

$q \to r$

$\therefore r$

Contradiction

$p \to F$

$\therefore \sim p$

Proof by contradiction

$p \to F$
$\therefore \sim p$

p	$p \to F$	$\sim p$	
T	F	F	
F	T	T	$\xleftarrow{\text{critical row}}$

3.9.1

(a) $t \to u$

(b) $p \vee \sim q$

(c) $p \to (u \to r)$

(d) q

(e) $\therefore t \to r$

3.9.2

(a) $p \to t$

(b) $\sim (q \to t) \to w$

(c) $p \vee q$

(d) $\sim w$

(e) $\therefore t$

3.9.8

(a) w

(b) $q \rightarrow r$

(c) $t \rightarrow s$

(d) $u \rightarrow s$

(e) $(\sim t \wedge \sim u) \rightarrow \sim w$

(f) $(s \vee y) \rightarrow (p \rightarrow q)$

(g) $\sim (p \rightarrow r) \vee x$

(h) $\therefore x$

3.9.9

(a) $p \to q$

(b) x

(c) $\sim (p \vee w) \to r$

(d) $q \to u$

(e) $x \to t$

(f) $w \to u$

(g) $r \vee s$

(h) $r \to F$

(i) $\therefore t \wedge s \wedge u$

3.9.10

(a) $u \rightarrow \sim p$

(b) $(\sim p \vee q) \rightarrow (r \rightarrow s)$

(c) $u \wedge \sim w$

(d) $t \rightarrow s$

(e) $(\sim t \wedge \sim r) \rightarrow w$

(f) $\therefore s$