Universal instantiation

$\forall\ x \in A,\ P(x)$
$a \in A$
$\therefore P(a)$

Universal modus ponens

$\forall\ x \in A,\ P(x) \rightarrow Q(x)$
$a \in A$
$P(a)$
$\therefore Q(a)$

Universal generalization

Suppose $a \in A$
$P(a)$
$\therefore \forall\ x \in A,\ P(x)$

Universal modus tollens

$\forall\ x \in A,\ P(x) \rightarrow Q(x)$
$a \in A$
$\sim Q(a)$
$\therefore \sim P(a)$

Existential Generalization

$a \in A$
$P(a)$
$\therefore \exists\ x \in A \mid P(x)$

Hypothetical division into cases

$p \vee q$
Suppose p
r
Suppose q
r
$\therefore r$

Existential instantiation

$\exists\ x \in A \mid P(x)$
Let $a \in A \mid P(a)$
$\therefore a \in A \wedge P(a)$

Hypothetical conditional

Suppose p
q
$\therefore p \rightarrow q$

3.14.4

a. $\forall\ x \in A,\ \ P(x) \wedge \sim Q(x)$

b. $\forall\ x \in A,\ \ x \in B$

c. $\forall\ x \in B,\ \ \sim Q(x) \rightarrow R(x)$

d. $\therefore \forall\ x \in A,\ \ R(x)$

3.14.5

a. $\forall\ x \in A,\ x \in B$
b. $\forall\ x \in B,\ \sim P(x)$
c. $\forall\ x \in A,\ Q(x) \to P(x)$
d. $\therefore \forall\ x \in A,\ \sim Q(x)$

(Extra # 1)

(a) $\forall\ y \in B, \exists\ x \in A \mid R(x, y)$

(b) $\forall\ x \in A, \forall\ y \in B, (P(x) \wedge R(x, y) \rightarrow Q(y))$

(c) $\therefore (\forall\ x \in A, P(x)) \rightarrow (\forall y \in B, Q(y))$

(Extra # 2)

(a) $\forall\ x \in A, P(x)$

(b) $\forall\ x \in A, x \in B \vee R(x)$

(c) $\forall\ y \in B, Q(y) \vee \sim P(y)$

(d) $\forall\ x \in A, R(x) \rightarrow Q(x)$

(e) $\therefore \forall x \in A, Q(x)$

(Extra # 3)

(a) $\forall\ x \in A, P(x) \to R(x)$

(b) $\exists\ x \in A \mid P(x)$

(c) $\forall\ x \in A, Q(x) \vee x \in Y$

(d) $\forall\ x \in A, P(x) \to \sim Q(x)$

(e) $\therefore \exists\ y \in B \mid R(y)$

3.14.10

a. $\forall\ x \in A,\ \exists\ y \in B \mid P(x, y)$

b. $\forall\ y \in B,\ \ Q(y) \vee R(y)$

c. $\forall\ x \in A,\ \ y \in B,\ \ P(x, y) \rightarrow \sim Q(y)$

d. $\exists\ x \in A \mid S(x)$

e. $\therefore \exists\ y \in B \mid R(y)$

3.14.11

a. $\forall\ x \in A,\ \ x \in B \wedge x \in C$
b. $\forall\ x \in C,\ \ x \in D \vee x \in E$
c. $\forall\ x \in B,\ \ x \in D \rightarrow P(x)$
d. $\forall\ x \in B,\ \ x \in E \rightarrow Q(x)$
e. $\forall\ x \in B,\ \ P(x) \vee Q(x) \rightarrow R(x)$
f. $\therefore \forall\ x \in A, R(x)$