If $x, y \in \mathbb{Z}$, then $x|y$ (that is, x *divides* y) if there exists $z \in \mathbb{Z}$ such that $x \cdot z = y$.

A *relation* R from a set X to a set Y is a set of ordered pairs from X and Y; it is a subset of $X \times Y$.

If R is a relation from a set X to a set Y, then the *image* of an element $x \in X$ under R is the set $\mathcal{I}_R(x) = \{y \in Y \mid (x, y) \in R\}$.

If R is a relation from a set X to a set Y, then the *image* of a set $A \subseteq X$ under R is the set $\mathcal{I}_R(A) = \{y \in Y \mid \exists\, a \in A \mid (a, y) \in R\}$.

If R is a relation from a set X to a set Y, then the *inverse* of R is the relation from Y to X defined by

$$R^{-1} = \{(y, x) \in Y \times X \mid (x, y) \in R\}$$

If R is a relation from a set X to a set Y and S is a relation from Y to a set Z, then the *composition* of R and S is the relation from X to Z defined by

$$S \circ R = \{(x, z) \in X \times Z \mid \exists\, y \in Y \text{ such that } (x, y) \in R \text{ and } (y, z) \in S\}$$

If R is a relation on a set X, then R is *reflexive* if $\forall\, x \in X, (x, x) \in R$.

If R is a relation on a set X, then R is *symmetric* if $\forall\, x, y \in X$, if $(x, y) \in R$ then $(y, x) \in R$.

If R is a relation on a set X, then R is *transitive* if $\forall\, x, y, z \in X$, if $(x, y) \in R$ and $(y, z) \in R$ then $(x, z) \in R$.