CSCI 243
Test 2 review

Quick summary: The test will probably consist in

- Two plain-old proof problems (Sections 4.(1-7))
- One powerset proof problem (Section 4.9)
- One problem that gives you a digraph of a relation and asks you to determine what properties that relation has and to identify the transitive closure and a topological sort. No proof. (Sections 5.(4-9))
- Two relation proof problems (Sections 5.(3-5))
- One "from theorems to algorithms" programming problem (Section 4.10)

The test will be held through Schoology. It will be made available at 2:00 (15 minutes before class officially starts) and must be finished by 4:30 (65 minutes after class officially ends). It shouldn't take that long, but that gives you a big margin on either side. If these times don't work for you (for example, because of time zone), let me know and I will make special arrangements for you.

4.2. Be prepared to write subset proofs using the definitions of intersection, union, Cartesian product, difference, and complement. The exercises in this section are good examples.

4.3. Be able to write proofs of set equality, similar to the subset proofs. The exercises in this section are good examples.

4.4. Be able to use proof by contradiction to show that a set is empty. The exercises in this section are good examples.

4.(5-7). Be able to write proofs of conditional and biconditional propositions. Be able to write proofs of propositions about integers. Know the definition of divides. The exercises in these sections are good examples.

4.9. Be able to use the definition of powerset in a proof. Theorem 4.6 and Exercises 4.9.(1-5) are good examples. You will not need to do anything based on the complicated sets in the discussion of Section 4.9 or like in Exercise 4.9.6. *The test will contain a problem asking you to prove a powerset result.*

4.10. Be able to write short ML functions based on information from theorems. *The test will contain a problem asking you to write an ML function or series of functions derived from a lemma, as in Exercises 4.10.(2–6). Exercises 4.10.(5 & 6) provide the best examples of what the test question will be like.*

General comment about relation problems (Chapter 5): I like to distinguish between "concrete problems" and "abstract problems." Concrete problems or propositions (and their proofs) are about specific relations. Examples of concrete relation problems from the exercises include 5.3.(1,2,3,6) and 5.4.(1–19). Abstract problems/propositions don't define or refer to a specific relation, but ask you to prove (or give a counterexample) about something for any relation R (or more than one relation) that has some specified properties. Examples of abstract relation problems include 5.3.(5, 7–11) and 5.4.(20–27).

5.3. Be able to prove things about relations (for example, whether two relations are equal) using the definitions of image, inverse, and composition. See Exercises 5.3.(5–11). (I will not ask you to "prove *or* give a counterexample" for problems **from this section**. I will tell you whether the proposition is true or false.)

5.4. Be able to prove that a relation is reflexive, symmetric, and/or transitive, as in the theorems and Exercises 5.4.(2–27). (It is possible that a question from this section will be a "prove or give a counterexample" question—that is, you may be required to determine which of the properties a relation has—but only for a concrete relation like $<$ or $|$, not for abstract relations like in 5.4.$(20 - 27)$.)

5.5. Be prepared to prove that a relation is an equivalence relation. Further, be able to prove other things about relations based on it being an equivalence relation or having one of the properties in Section 5.4. See the theorems and exercises.

There will not be anything on the test about the relation induced by a partition or about equivalence classes.

5.6&7. Know what the transitive closure is. Be able to identify the transitive closure of a given relation (for example, if I asked what the transitive closure of "is one less than" is, the answer would be $<$.) *The test probably will contain a problem asking you what the transitive closure of a specific relation is. You will need to give an informal description of the specific relation that is the transitive closure of the given relation.*

5.8&9. Know what partial orders, total orders, and topological sorts are. Be able to give an example partial order for a set and an example topological sort for a partial order.

The test will include a "cheat sheet" giving you the formal definitions of divides, image, composition, inverse, identity relation, reflexivity, symmetry, transitivity, and antisymmetry. You should still have these definitions "mostly memorized." Do not be dependent on the cheat sheet—use it only to get past a mental block.