Computer Science 345
Practice problems for test 1

Consider this implementation of the `size()` method for a bit vector implementation of an NSet:

```
public class BitVecNSet implements NSet {
    private byte[] internal;  /** The array of bytes, used as a bit vector. */
    private int range;    /** One greater than the largest number than can be stored. */
    ...

    public int size() {
        int count = 0;
        for (int i = 0; i < internal.length - 1; i++) { // first, outer loop
            byte current = internal[i];
            for (int j = 0; j < 8; j++) {      // second, inner loop
                count += current & 1;
                current >>= 1;
            }
        }
        byte current = internal[internal.length - 1];
        for (int j = 0; j < range % 8; j++) {   // third loop
            count += current & 1;
            current >>= 1;
        }
        return count;
    }
}
```

Let N be the size of the universe the set is drawn from (note `range` $= N$) and let A be the conceptual set.

1. Write a loop invariant for the first, outer for loop that explains what `count` is and relates it to `i`, `internal`, and/or the conceptual set.

2. Write a loop invariant for the inner for loop that explains `current` and relates `current` to `count`, `i`, `j`, `internal`, and/or the conceptual set.

(You are not asked to do anything with the third loop)

3. Consider this implementation of the `decreaseKeyAt()` method for the implementation of a heap:

```
public class IntHeap {

    /**
     * The array containing the internal data of the heap.
     */
    private int internal[];

    ...

    protected void decreaseKeyAt(int i) {

        int lIndex = left(i);
        int rIndex = right(i);
        while (lIndex < heapSize &&
                (internal[i] < internal[lIndex]) ||
                        (rIndex < heapSize &&
                                internal[i] < internal[rIndex])) {
            int greatestIndex = internal[i] < internal[lIndex] ?
                    lIndex : i;
            greatestIndex =
                    rIndex < heapSize && internal[greatestIndex] < internal[rIndex] ?
                            rIndex : greatestIndex;
            int temp = internal[i];
            internal[i] = internal[greatestIndex];
            internal[greatestIndex] = temp;
            i = greatestIndex;
            lIndex = left(i);
            rIndex = right(i);
        }

    }
}
```

Write a precondition for this method and an invariant for the loop that explains how the variable `i` is used, specifically what assumptions are made and maintained about the heap in relation to index `i`.

4. What is the running time of counting sort (as a big-Oh category) on an array of size n with values in the range $[0, m)$ for some whole number m?

5. What is an *abstract data type*? Specifically, what makes it *abstract*?

6. How do lists and stacks differ? How do sets and maps differ? For any two distinct ADTs x and y that we have defined, how to x and y differ? How do they all differ from the data structures we have studied?

7. If you needed to write a quick implementation of set and had implementations of list, map, and bag at your disposal, which would you use, and what would be the key insight of your strategy? (Recommended: Write out the code for an implementation of `(Whatever)Set`, and be able to articulate why you chose whatever and how you're using it.)

8. Give one advantage each for array-based implementations and linked implementations, in comparison with each other.

9. Explain a circumstance in which an adjacency-matrix implementation has some advantages over adjacency-list.

10. Suppose we have the following interface for graphs, where vertices are identified by number:

```
public interface Graph {
    int numVertices();
    // return the vertices adjacent to v
    Iterable<Integer> adjacents(int v);
}
```

Suppose further you have a class that implements the `Graph` interface above as a *directed* graph, with the following public signatures:

```
public class DirectedGraph implements Graph {
    public DirectedGraph(int numVertices);
    public int numVertices();
    public Iterable<Integer> adjacents(int v);
    // add an edge from u to v
    public void addEdge(int u, int v);
}
```

Note that the constructor takes only the *number of vertices*; edges are added later using the `addEdge()` method.

Write a class that implements the `Graph` interface above as an *undirected* graph, reusing the class `DirectedGraph`. Like `DirectedGraph`, your class should have a constructor that takes only the number of vertices and a method `void addEdge(int u, int v)`.

11. Suppose we have the following interface for graphs, where vertices are identified by number:

```
public interface Graph {
    int numVertices();
    // return the vertices adjacent to v
    Iterable<Integer> adjacents(int v);
}
```

One can iterate over all edges of graph `g` with the following nested loops:

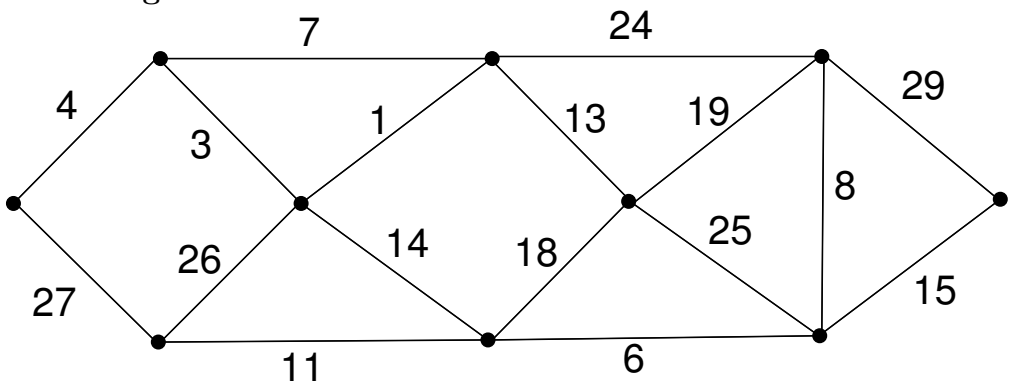
```
for (int u = 0; u < g.numVertices; u++)
    for (int v : g.adjacents(u))
        // do something with edge (u, v)
```

Assume the body of the inner loop is constant.

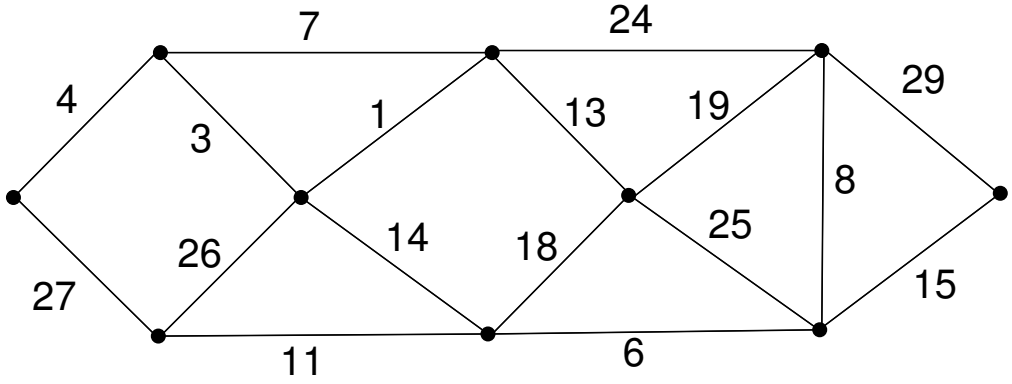
a. What is the running time for this code as a big-oh category in terms of V and E if `g` is implemented using adjacency lists?

b. What is the running time for this code as a big-oh category in terms of V and E if `g` is implemented using an adjacency matrix?

12. In the following graph, highlight the edges that form a minimum spanning tree and label them to indicate an order in which they would be added to the tree **using Kruskal's algorithm**.



13. Using the same graph as before, highlight the edges that form a minimum spanning tree and label them to indicate an order in which they would be added to the tree **using Prim's algorithm**.



14. Explain how Dijkstra's algorithm finds an SSSP tree in one round of relaxations.

```
public interface List<E>  {
    E get(int index);
    int size();
}

public class ArrayList implements List<String> {

    private String[] internal;

    public ArrayList(String[] items) {
        internal = new String[items.length];
        for (int i = 0; i < items.length; i++)
            internal[i] = items[i];
    }

    public String get(int index) {
        return internal[index];
    }

    public int size() { return internal.length; }

}

public class LinkedList implements List<String> {

    private class Node{
        String datum;
        Node next;
        public Node(String datum, Node next) {
            this.datum = datum;
            this.next = next;
        }
    }

    Node head;

    public LinkedList(String[] items) {
        head = null;
        for (int i = items.length - 1; i >= 0; i--)
            head = new Node(items[i], head);
    }

    public String get(int index) {
        Node current = head;
        for (int i = 0; i < index; i++) current = current.next;
        return current.datum;
    }

    public int size() {
        int count = 0;
        for (Node current = head; current != null;  current = current.next)
            count++;
        return count;
    }
}
```