Chapter 3 roadmap:

- Propositions, boolean logic, logical equivalences. **Game 1** (last week Friday)
- Conditional propositions. **SML** (Monday)
- Arguments. **Game 2** (Today)
- Predicates and quantification. **SML** (Friday)
- Quantified arguments. **Game 3** (Next week Monday)
- Review for test. (Next week Wednesday)

Today:

- Define arguments
- Consider known "syllogisms"
- Practice verifying argument forms (Game 2)

Valid argument

If it is Monday, then it is raining
It is Monday.
Therefore it is raining.

$p \rightarrow q$
p
$\therefore q$

		premise	*premise*	*conclusion*
p	q	$p \rightarrow q$	q	
T	T	T	T	← *critical row*
T	F	F	F	
F	T	T	T	
F	F	T	F	

Invalid argument

If it is raining, then there are clouds
There are clouds.
Therefore it is raining.

$p \rightarrow q$
q
$\therefore p$

p	q	$p \rightarrow q$	p	
T	T	T	T	← *critical row*
T	F	F	T	
F	T	T	F	← *critical row*
F	F	T	F	

premise   *premise*   *conclusion*

Alternate definition of validity

Valid argument

p	q	$p \to q$	$(p \land (p \to q)) \to q$
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	T

Invalid argument

p	q	$p \to q$	$(q \land (p \to q)) \to p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

Modus tollens

If it is spring, then the daffodils bloom.
The daffodils aren't blooming.
Therefore it is not spring.

p	q	$p \rightarrow q$	$\sim q$	$\sim p$
T	T	T	F	
T	F	F	T	
F	T	T	F	
F	F	T	T	T

Modus Ponens
$p \rightarrow q$
p
$\therefore q$

Modus Tollens
$p \rightarrow q$
$\sim q$
$\therefore \sim p$

Generalization
p
$\therefore p \vee q$

Specialization
$p \wedge q$
$\therefore p$

Elimination
$p \vee q$
$\sim p$
$\therefore q$

Transitivity
$p \rightarrow q$
$q \rightarrow r$
$\therefore p \rightarrow r$

Division into cases
$p \vee q$
$p \rightarrow r$
$q \rightarrow r$
$\therefore r$

Contradiction
$p \rightarrow F$
$\therefore \sim p$

Proof by contradiction

$p \to F$
$\therefore \sim p$

p	$p \to F$	$\sim p$
T	F	F
F	T	T	$\xleftarrow{\textit{critical row}}$

Mod Pon	Mod Tol	Generalization	Specialization	Elimination	Transitivity	Div into cases	Contradiction
$p \to q$	$p \to q$	p	$p \wedge q$	$p \vee q$	$p \to q$	$p \vee q$	$p \to F$
p	$\sim q$	$\therefore p \vee q$	$\therefore p$	$\sim p$	$q \to r$	$p \to r$	$\therefore \sim p$
$\therefore q$	$\therefore \sim p$			$\therefore q$	$\therefore p \to r$	$q \to r$	
						$\therefore r$	

3.9.1

(a) $t \to u$

(b) $p\vee \sim q$

(c) $p \to (u \to r)$

(d) q

(e) $\therefore t \to r$

Mod Pon	Mod Tol	Generalization	Specialization	Elimination	Transitivity	Div into cases	Contradiction
$p \rightarrow q$	$p \rightarrow q$	p	$p \wedge q$	$p \vee q$	$p \rightarrow q$	$p \vee q$	$p \rightarrow F$
p	$\sim q$	$\therefore p \vee q$	$\therefore p$	$\sim p$	$q \rightarrow r$	$p \rightarrow r$	$\therefore \sim p$
$\therefore q$	$\therefore \sim p$			$\therefore q$	$\therefore p \rightarrow r$	$q \rightarrow r$	
						$\therefore r$	

3.9.2

(a) $p \rightarrow t$

(b) $\sim (q \rightarrow t) \rightarrow w$

(c) $p \vee q$

(d) $\sim w$

(e) $\therefore t$

Mod Pon	Mod Tol	Generalization	Specialization	Elimination	Transitivity	Div into cases	Contradiction
$p \rightarrow q$	$p \rightarrow q$	p	$p \wedge q$	$p \vee q$	$p \rightarrow q$	$p \vee q$	$p \rightarrow F$
p	$\sim q$	$\therefore p \vee q$	$\therefore p$	$\sim p$	$q \rightarrow r$	$p \rightarrow r$	$\therefore \sim p$
$\therefore q$	$\therefore \sim p$			$\therefore q$	$\therefore p \rightarrow r$	$q \rightarrow r$	
						$\therefore r$	

3.9.8

(a) w

(b) $q \rightarrow r$

(c) $t \rightarrow s$

(d) $u \rightarrow s$

(e) $(\sim t \wedge \sim u) \rightarrow \sim w$

(f) $(s \vee y) \rightarrow (p \rightarrow q)$

(g) $\sim (p \rightarrow r) \vee x$

(h) $\therefore x$

Mod Pon	Mod Tol	Generalization	Specialization	Elimination	Transitivity	Div into cases	Contradiction
$p \to q$	$p \to q$	p	$p \land q$	$p \lor q$	$p \to q$	$p \lor q$	$p \to F$
p	$\sim q$	$\therefore p \lor q$	$\therefore p$	$\sim p$	$q \to r$	$p \to r$	$\therefore \sim p$
$\therefore q$	$\therefore \sim p$			$\therefore q$	$\therefore p \to r$	$q \to r$	
						$\therefore r$	

3.9.9

(a) $p \to q$

(b) x

(c) $\sim (p \lor w) \to r$

(d) $q \to u$

(e) $x \to t$

(f) $w \to u$

(g) $r \lor s$

(h) $r \to F$

(i) $\therefore t \land s \land u$

Mod Pon	Mod Tol	Generalization	Specialization	Elimination	Transitivity	Div into cases	Contradiction
$p \rightarrow q$	$p \rightarrow q$	p	$p \wedge q$	$p \vee q$	$p \rightarrow q$	$p \vee q$	$p \rightarrow F$
p	$\sim q$	$\therefore p \vee q$	$\therefore p$	$\sim p$	$q \rightarrow r$	$p \rightarrow r$	$\therefore \sim p$
$\therefore q$	$\therefore \sim p$			$\therefore q$	$\therefore p \rightarrow r$	$q \rightarrow r$	
						$\therefore r$	

3.9.10

(a) $u \rightarrow \sim p$

(b) $(\sim p \vee q) \rightarrow (r \rightarrow s)$

(c) $u \wedge \sim w$

(d) $t \rightarrow s$

(e) $(\sim t \wedge \sim r) \rightarrow w$

(f) $\therefore s$

For next time:

Pg 119: 3.8.(3 & 5)
Pg 122: 3.9.(3-7)

Read carefully 3.(10 & 11)
Skim 3.(12 & 13)

(No quiz)