"Stand-alone topics" outline:

- ▶ Foldl, and how to model mathematical functions (Last Friday)
- ▶ Fixed-point iteration (**Today**)
- ▶ The Huffman encoding (Wednesday)
- ▶ Review for the final exam (Friday)

Today:

- ▶ Currying functions
- ▶ Specific problem: calculating square roots
- ▶ Medium-general problem: finding roots of real-valued functions
- ▶ Very general solution: Fixed-point iteration
- ▶ Applying general solution to specific problems

arity, n The number of elements by virtue of which something is unary, binary, etc.
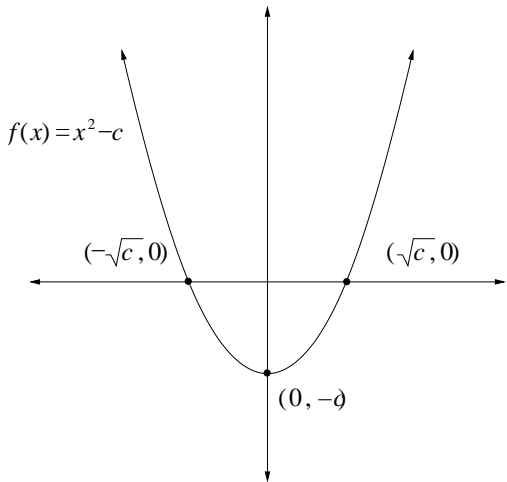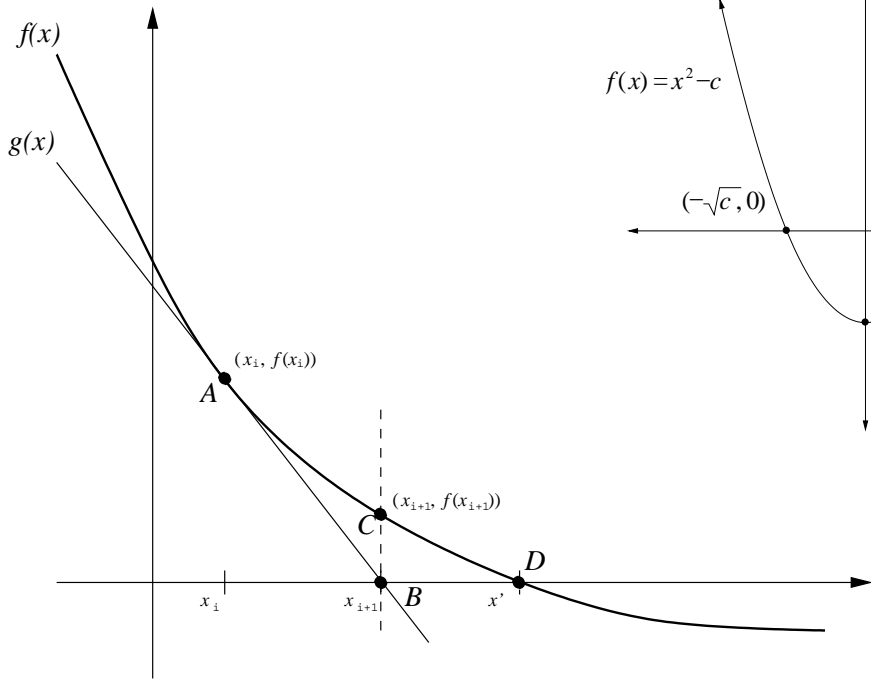
curry, v.1 (a) To rub down or dress (a horse etc.) with a comb. (b) (Applied to persons) to seek to win favor, or ingratiate oneself with another.

curry, v.2 *Obsolete.* To ride or run with haste or rapidity

curry, v.3 To flavor or prepare with curry or curry-powder.

curry, v.4 To transform a function that takes multiple arguments into a function that takes a single argument and returns a new function that takes the remainder of the arguments and returns the result.

$f(x)$

$g(x)$

$(x_i, f(x_i))$

A

$(x_{i+1}, f(x_{i+1}))$

C

D

x_i

x_{i+1}

B

x'

$f(x) = x^2 - c$

$(-\sqrt{c}, 0)$

$(\sqrt{c}, 0)$

$(0, -c)$

Point-slope form: Given a point (x_1, y_1) and a slope m,

$$
\begin{aligned}
y - y_1 &= m \cdot (x - x_1) \\
y &= m \cdot (x - x_1) + y_1
\end{aligned}
$$

To find **root** of function f near a guess x_i, draw a tangent line at $(x_1, f(x_1))$:

$$g(x) = f'(x_1) \cdot (x - x_1) + f(x_1)$$

Then find where this line hits the x-axis:

$$
\begin{aligned}
0 &= f'(x_1)(x - x_1) + f(x_1) \\
x &= \frac{x_1 f'(x_1) - f(x_1)}{f'(x_1)} \\
&= x_1 - \frac{f(x_1)}{f'(x_1)}
\end{aligned}
$$

Consider that our *next guess*. In general,

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

In general, to find a *new, improved guess* x_{i+1} for a root of function f from current guess x_i,

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

To find $\sqrt{c}$, apply this to $f(x) = x^2 - c$.

$$f'(x) = 2x$$

And so the *guess-improving function* is

$$I(x) = x - \frac{x^2 - c}{2x}$$

Algorithm for finding an approximation for a root:

If our guess is good enough, then stop.
Otherwise, improve it and try again.

Fixed-point iteration. Algorithm for any iterative guess-improving process:

If our guess is good enough, then stop.
Otherwise, improve it and try again.

Elements:

- ▶ Guess-improver
- ▶ Current guess
- ▶ Good-enough tester

For finding roots using Newton's method, the guess-improver is

$$G(x) = x - \frac{f(x)}{f'(x)}$$

What if x is an exact root, that is, if $f(x) = 0$?

Lessons:

▶ Interaction between mathematics and computer science: An algorithm solves a mathematical problem.

▶ Interaction between continuous and discrete mathematics: The *continuous* functions are themselves *discrete* objects.

▶ Generalization in math and computer science: Functions and algorithms can be parameterized to become more general.

For next time:

    Pg 375: 7.13.(1,3)

    Skim 6.12, new version found on Schoology