

Where we are:

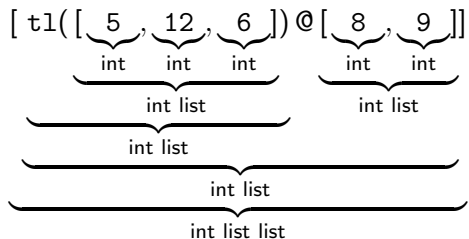
- ▶ More about functions in SML; introduction to lists (this past Friday)
- ▶ Functions on lists; powersets (**today**)
- ▶ Application: A language processor (Wednesday)
- ▶ Propositional forms, logical equivalence [Start Chapter 3] (Friday)

Today:

- ▶ Functions on lists
 - ▶ Examples
 - ▶ Principles
 - ▶ Practice
- ▶ Powersets
 - ▶ Definition
 - ▶ Exploration

Review

- ▶ List literals: `[1, 4, 12, 3]`, `[]`
- ▶ Analytic operations: `hd`, `tl`
- ▶ Synthetic operations: `::` (`cons`), `@` (`cat`)
- ▶ Lists vs tuples
- ▶ Type analysis problems :



- ▶ Lists as models for sets

Powersets

- ▶ Informal definition: The powerset of a set is the set of all subsets of that set.
- ▶ Formal definition: The powerset of a set X is

$$\mathcal{P}(X) = \{ Y \mid Y \subseteq X \}$$

- ▶ For “set of sets,” think “box of boxes.”
- ▶ Examples:

Why powersets seem to throw some people:

- ▶ The elements of a powerset are themselves sets.
- ▶ Suppose $X \subseteq \mathcal{U}$. Then
 - ▶ If $x \in X$, then $x \in \mathcal{U}$
 - ▶ $\mathcal{P}(X) \not\subseteq \mathcal{U}$, but rather $\mathcal{P}(X) \subseteq \mathcal{P}(\mathcal{U})$
 - ▶ If $A \in \mathcal{P}(X)$, then $A \in \mathcal{P}(\mathcal{U})$
- ▶ $\emptyset \neq \{\emptyset\}$. $|\emptyset| = 0$, but $|\{\emptyset\}| = 1$

Which are true?

$$\{3\} \in \mathcal{P}(\{1, 2, 3, 4, 5\})$$

$$3 \in \mathcal{P}(\{1, 2, 3, 4, 5\})$$

$$\{3\} \subseteq \mathcal{P}(\{1, 2, 3, 4, 5\})$$

$$3 \subseteq \mathcal{P}(\{1, 2, 3, 4, 5\})$$

$$a \in A \text{ iff } \{a\} \in \mathcal{P}(A)$$

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$$a \in A \text{ iff } \{a\} \subseteq \mathcal{P}(A)$$

$$A \subseteq B \text{ iff } A \in \mathcal{P}(B)$$

Which are true?

$$A \subseteq B \text{ iff } A \subseteq \mathcal{P}(B)$$

$$\{A\} \subseteq \mathcal{P}(A)$$

$$A \in \mathcal{P}(A)$$

$$\{A\} \in \mathcal{P}(A)$$

$$\mathbb{Z} \in \mathcal{P}(\mathbb{R})$$

$$\emptyset = \mathcal{P}(\emptyset)$$

Note that

- ▶ $a \in A$ iff $\{a\} \in \mathcal{P}(A)$
- ▶ $A \subseteq B$ iff $A \in \mathcal{P}(B)$
- ▶ $A \subseteq B$ iff $\mathcal{P}(A) \subseteq \mathcal{P}(B)$
- ▶ $\mathcal{P}(\emptyset) = \{\emptyset\} \neq \emptyset$

Observe

$$\begin{aligned}\mathcal{P}(\{1, 2, 3\}) &= \{ \emptyset \\ &\quad \{1\}, \{2\}, \{3\} \\ &\quad \{1, 2\}, \{1, 3\}, \{2, 3\} \\ &\quad \{1, 2, 3\} \} \\ &= \{ \{1\}, \{1, 2\}, \{1, 3\}, \{1, 2, 3\} \\ &\quad \emptyset, \{2\}, \{3\}, \{2, 3\} \} \\ &= \mathcal{P}(\{2, 3\}) \cup \left[\begin{array}{l} \text{1 added to each set} \\ \text{of } \mathcal{P}(\{2, 3\}) \end{array} \right] = \mathcal{P}(\{2, 3\}) \cup \\ &\quad \{ \{1\} \cup X \mid X \in \mathcal{P}(\{2, 3\}) \}\end{aligned}$$

If $a \in A$, then $\mathcal{P}(A) = \mathcal{P}(A - \{a\}) \cup \{ \{a\} \cup X \mid X \in \mathcal{P}(A - \{a\}) \}$

What is $|\mathcal{P}(X)|$ in terms of $|X|$?

For next time:

If you had trouble on the programming problems from last time, ask for help and try again.

Pg 74: 2.2.(2, 3, 8, 9, 11, 13, 15)

See notes on Ex 2.2.8 and 2.2.9 on the Schoology description of the assignment. See also the code from class for “starter code.”

Review 2.4 as necessary

Read carefully 2.5

Skim 2.6