Chapter 3 roadmap:

- Propositions, boolean logic, logical equivalences. **Game 1** (last week Friday)
- Conditional propositions. **SML** (Monday)
- Arguments. **Game 2** (Wednesday)
- Predicates and quantification. **SML** (Today)
- Quantified arguments. **Game 3** (Next week Monday)
- Review for test. (Next week Wednesday)
- Test 1. (Next week Friday)

Today:

- Predicates
- Quantification
- Practice quantification using programming problems

Project proposal due Friday, Feb 18.

Propositions:

- $3 < 5$
- It's Thursday and it is snowing.
- If $3 < 5$ then $12 < 67$.

Propositional forms:

- $p \wedge q$
- $p \rightarrow q$

Four ways to interpret/define the idea of a *predicate*

- A predicate is a proposition with a parameter.

$$x < 5 \qquad\qquad x \text{ is orange}$$

- A predicate is a function whose value is true or false.

$$P(x) = x < 5 \qquad\qquad Q(x) = x \text{ is orange}$$

- A predicate is a part of a sentence that complements a noun phrase to make a proposition.
  A pumpkin is orange.
- A predicate is a truth set

	$P : \mathbb{N} \to \mathbb{B}, P(x) = x < 5$	$Q(x) = x$ is orange
Truth set:	$\{1, 2, 3, 4\}$	{ pumpkin, fall leaves, orange juice, ... }

Universal quantification

"For all multiples of 3, the sum of their digits is a multiple of 3."

Let D be the set of multiples of 3, that is
$D = \{n \in \mathbb{N} \mid n \texttt{ mod } 3 = 0\} = \{3, 6, 9, 12, 15, 18, \ldots\}$

$$\forall\ x \in D, \texttt{sum}(\texttt{digify}(x)) \in D$$

Other examples:

- $\forall x \in \{5, 7, 19, 23, 43\}$, x is prime.
- $\forall x \in \{4, 16, 25, 31\}$, x is a perfect square.

Existential quantification

“There is a multiple of 3 that is not a perfect square.”

$$\exists\ x \in D \mid x \text{ is not a perfect square}$$

Alternately, “Some multiples of 3 are not perfect squares.”

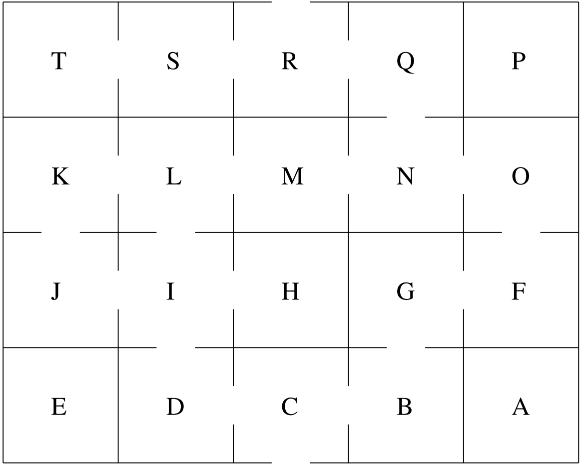
General forms for universal and existential quantification:

$$\forall\ x \in X, P(x) \qquad\qquad \exists\ x \in X \mid P(x)$$

$\forall\ x \in \emptyset, P(x)$ is **always (vacuously) true**.

$\exists\ x \in \emptyset \mid P(x)$ is **always false**

$$
\begin{aligned}
&\sim (\forall\ x \in X, P(x)) \\
&\quad \equiv \quad \sim (P(x_1) \wedge P(x_2) \wedge \cdots) \\
&\quad \equiv \quad \sim P(x_1) \vee \sim P(x_2) \vee \cdots \quad \text{By DeMorgan's Law} \\
&\quad \equiv \quad \exists\ x \in X \mid\ \sim P(x)
\end{aligned}
$$



1. Bob passed through P.
2. Bob passed through N.
3. Bob passed through M.
4. If Bob passed through O, then Bob passed through F.
5. If Bob passed through K, then Bob passed through L.
6. If Bob passed through L, then Bob passed through K.

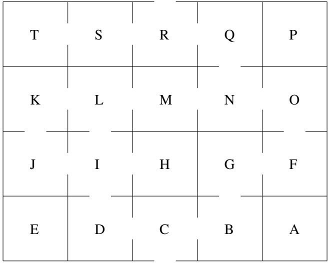
Based on example by Susanna Epp, 2006

Let X be the routes through the maze, that is,
$X = \{CBGFONQR, CDILMNQR, CDIJKLMNQR\}$

Let $P(x) =$ route x contains L,
$Q(x) =$ route x contains K.

Consider $\forall\ x \in X, P(x) \to Q(x)$.

X	$P(x)$	$Q(x)$	$P(x) \to Q(x)$
$CBGFONQR$			
$CDILMNQR$			
$CDIJKLMNQR$			



For next time:

Pg 133: 3.12.(1 & 2)
Pg 135: 3.13.(4 & 5)

Read 3.14