Universal instantiation

$\forall\ x \in A,\ P(x)$
$a \in A$
$\therefore P(a)$

Universal modus ponens

$\forall\ x \in A,\ P(x) \to Q(x)$
$a \in A$
$P(a)$
$\therefore Q(a)$

Universal generalization

Suppose $a \in A$
$P(a)$
$\therefore \forall\ x \in A,\ P(x)$

Universal modus tollens

$\forall\ x \in A,\ P(x) \to Q(x)$
$a \in A$
$\sim Q(a)$
$\therefore \sim P(a)$

Existential instantiation

$\exists\ x \in A \mid P(x)$
Let $a \in A \mid P(a)$
$\therefore a \in A \wedge P(a)$

Existential Generalization

$a \in A$
$P(a)$
$\therefore \exists\ x \in A \mid P(x)$

Hypothetical conditional

Suppose p
q
$\therefore p \to q$

Hypothetical division into cases

$p \vee q$
Suppose p
r
Suppose q
r
$\therefore r$

(Extra # 2)

(a) $\forall\ x \in A, P(x)$

(b) $\forall\ x \in A, x \in B \vee R(x)$

(c) $\forall\ y \in B, Q(y) \vee \sim P(y)$

(d) $\forall\ x \in A, R(x) \rightarrow Q(x)$

(e) $\therefore \forall x \in A, Q(x)$

	Suppose $a \in A$	
(i)	$a \in B \wedge R(a)$	by supposition, (b), and UI
	Suppose $a \in B$	
(ii)	$Q(a) \vee \sim P(a)$	by supposition, (c), and UI
(iii)	$P(a)$	by supposition, (a), and UI
(iv)	$Q(a)$	by (ii), (iii), and elimination
	Suppose $R(a)$	
(v)	$Q(a)$	by supposition, (c), and UMP
(vi)	$Q(a)$	by (i), (iv),(v), and HDC
(vii)	$\therefore \forall x \in A, Q(x)$	by supposition, (vi), and UG

(Extra # 3)

(a) $\forall\ x \in A, P(x) \rightarrow R(x)$

(b) $\exists\ x \in A \mid P(x)$

(c) $\forall\ x \in A, Q(x) \vee x \in B$

(d) $\forall\ x \in A, P(x) \rightarrow \sim Q(x)$

(e) $\therefore \exists\ y \in B \mid R(y)$

	Let $a \in A \mid P(a)$	
(i)	$a \in A \wedge P(a)$	By (b) and EI
(ii)	$a \in A$	By (i) and specialization
(iii)	$P(a)$	By (i) and specialization
(iv)	$\sim Q(a)$	by (ii), (iii), (d), and UMP
(v)	$Q(a) \vee a \in B$	by (ii), (c), and UI
(vi)	$a \in B$	by (iv), (v), and elimination
(vii)	$R(a)$	by (ii), (iii), (a), and UMP
(viii)	$\therefore \exists\ y \in B \mid R(y)$	by (vi), (vii), and EG

3.14.10

(a) $\forall\ x \in A,\ \exists\ y \in B \mid P(x,y)$

(b) $\forall\ y \in B,\ \ Q(y) \vee R(y)$

(c) $\forall\ x \in A,\ \ y \in B,$
$P(x,y) \rightarrow \sim Q(y)$

(d) $\exists\ x \in A \mid S(x)$

(e) $\therefore \exists\ y \in B \mid R(y)$

	Let $a \in A \mid S(a)$	
(i)	$a \in A \wedge S(a)$	by (d) and EI
(ii)	$a \in A$	by (i) and specialization
(iii)	$\exists\ y \in B \mid P(a,y)$	by (ii), (a), and UI
	Let $b \in B \mid P(a,b)$	
(iv)	$b \in B \wedge P(a,b)$	by (iii) and EI
(v)	$b \in B$	by (iv) and specialization
(vi)	$P(a,b)$	by (iv) and specialization
(vii)	$\forall\ y \in B, P(a,y) \rightarrow \sim Q(y)$	by (c), (ii), UI
(viii)	$\sim Q(b)$	by (vii), (v), and UMP
(ix)	$Q(b) \vee R(b)$	by (b), (v), and UI
(x)	$R(b)$	by (ix), (vii), and elimination
(xi)	$\therefore\ \exists\ y \in B \mid R(y)$	by (v), (x), and EG

3.14.11

(a) $\forall\, x \in A,\ x \in B \wedge x \in C$

(b) $\forall\, x \in C,\ x \in D \vee x \in E$

(c) $\forall\, x \in B,\ x \in D \to P(x)$

(d) $\forall\, x \in B,\ x \in E \to Q(x)$

(e) $\forall\, x \in B,\ (P(x) \vee Q(x)) \to R(x)$

(f) $\therefore \forall\, x \in A, R(x)$

	Suppose $a \in A$	
(i)	$a \in B \wedge a \in C$	by supposition, (a), and UI
(ii)	$a \in B$	by (i) and specialization
(iii)	$a \in C$	by (i) and specialization
(iv)	$a \in D \vee a \in E$	by (iii),(b), and UI
	Suppose $a \in D$	
(v)	$P(a)$	by (ii), supposition, (c), and UMP
(vi)	$P(a) \vee Q(a)$	by (v) and (plain old) generalization
	Suppose $a \in E$	
(vii)	$Q(a)$	by (ii), supposition, (d), and UMP
(viii)	$P(a) \vee Q(a)$	by (vii) and (plain old) generalization
(ix)	$P(a) \vee Q(a)$	by (iv), (vi), (viii), and HDC
(x)	$R(a)$	by supposition, (ix), (e), and UMP
(xi)	$\therefore \forall\, x \in A, R(x)$	by supposition, (x), and UG.

Which of the following are true?

$$-((x-y)+(x-z)) \quad = \quad -(x-y)-(x-z)$$

$$-((x-y)+(x-z))\cdot z \quad = \quad -(x-y)-(x-z)\cdot z$$

$$\sim(p\wedge q) \quad \equiv \quad \sim p\vee\sim q$$

$$\sim(p\wedge q)\wedge r \quad \equiv \quad \sim p\vee\sim q\wedge r$$

Which of the following are true?

$$
\begin{aligned}
(x + y) + z &= x + (y + z) \\
(x - y) + z &= x - (y + z) \\
(p \vee q) \vee r &\equiv p \vee (q \vee r) \\
(p \vee q) \wedge r &\equiv p \vee (q \wedge r)
\end{aligned}
$$

1. Write a function `leastSigDigs` that takes a list of ints and returns a list of the least significant digits in those lists. For example, `leastSigDigs[283, 7234, 5, 2380]` would return `[3, 4, 5, 0]`.

2. Write a function `hasEmpty` that takes a list of lists (of any type) and determines whether or not the list of lists contains an empty list. For example, `hasEmpty([[1,2,3], [4,5], [], [6,7]])` would return `true`.