Welcome

CSCI 243
Wheaton College
Thomas VanDrunen
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What is this course about?

Proof-based **discrete mathematics**
and
programming in the **functional style**

. . . with three audiences in mind:

- Computer science majors
- Math majors
- Everyone else

Goals and themes

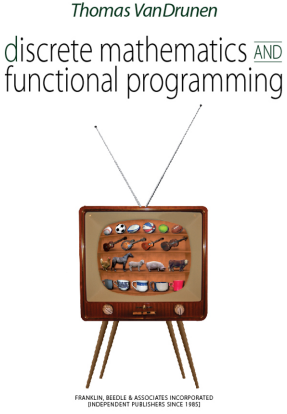
At the end of this course you should be able to

- Manipulate symbolic logical forms
- Write mathematical proofs, especially for results from basic set theory
- Write simple programs in the SML programming language

Throughout this course, we will see these recurring themes:

- Formal definitions
- Recursive thinking
- Analysis and synthesis

Textbook



Discrete Mathematics and Functional Programming

5 is a natural number; *or* the collection of natural numbers contains 5.

Adding 0 to the collection of natural numbers makes the collection of whole numbers.

Merging the algebraic numbers and the transcendental numbers makes the real numbers.

Transcendental numbers are those real numbers which are not algebraic numbers.

Nothing is both transcendental and algebraic, *or* the collection of things both transcendental and algebraic is empty.

Negative integers are both negative and integers.

All integers are rational numbers.

Since all rational numbers are algebraic numbers and all algebraic numbers are real numbers, it follows that all rational numbers are real numbers.

Axiom (Existence.)

There is a set with no elements.

Axiom (Extensionality.)

If every element of a set X is an element of a set Y and every element of Y is an element of X, then $X = Y$.

(Exercises 1.3.(1–10).)

-12 $\in \mathbb{N}$.

-12 $\in \mathbb{W}$.

-12 $\in \mathbb{Z}$.

-12 $\in \mathbb{Q}$.

-12 $\in \mathbb{R}$.

$\frac{1}{56} \in \mathbb{N}$.

$\frac{1}{56} \in \mathbb{W}$.

$\frac{1}{56} \in \mathbb{Z}$.

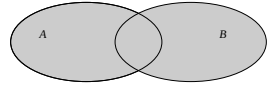
$\frac{1}{56} \in \mathbb{Q}$.

$\frac{1}{56} \in \mathbb{R}$.

Union

$A \cup B = \{ x \mid x \in A \text{ or } x \in B \}$

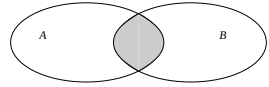
$$\begin{aligned} \{1,2,3\} \cup \{2,3,4\} &= \{1,2,3,4\} \\ \{1,2\} \cup \{3,4\} &= \{1,2,3,4\} \\ \{1,2\} \cup \{1,2,3\} &= \{1,2,3\} \end{aligned}$$



Intersection

$A \cap B = \{ x \mid x \in A \text{ and } x \in B \}$

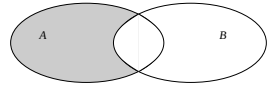
$$\begin{aligned} \{1,2,3\} \cap \{2,3,4\} &= \{2,3\} \\ \{1,2\} \cap \{3,4\} &= \emptyset \\ \{1,2\} \cap \{1,2,3\} &= \{1,2\} \end{aligned}$$



Difference

$A - B = \{ x \mid x \in A \text{ and } x \notin B \}$

$$\begin{aligned} \{1,2,3\} - \{2,3,4\} &= \{1\} \\ \{1,2\} - \{3,4\} &= \{1,2\} \\ \{1,2\} - \{1,2,3\} &= \emptyset \end{aligned}$$



(Exercises 1.4.(11–18).)

$-12 \in \mathbb{R}^-$.

$\mathbb{A} \subseteq \mathbb{C}$.

$\mathbb{R} \subseteq \mathbb{C} \cap \mathbb{R}^-$

$4 \in \mathbb{C}$.

$\mathbb{Q} \cap \mathbb{T} = \emptyset$.

$\frac{1}{63} \in \mathbb{Q} - \mathbb{R}$.

$\mathbb{Z} - \mathbb{R}^- = \mathbb{W}$.

$\mathbb{T} \cup \mathbb{Z} \subseteq \mathbb{A}$.

For next time:

Review 1.(1-2)

Read 1.(3-5)