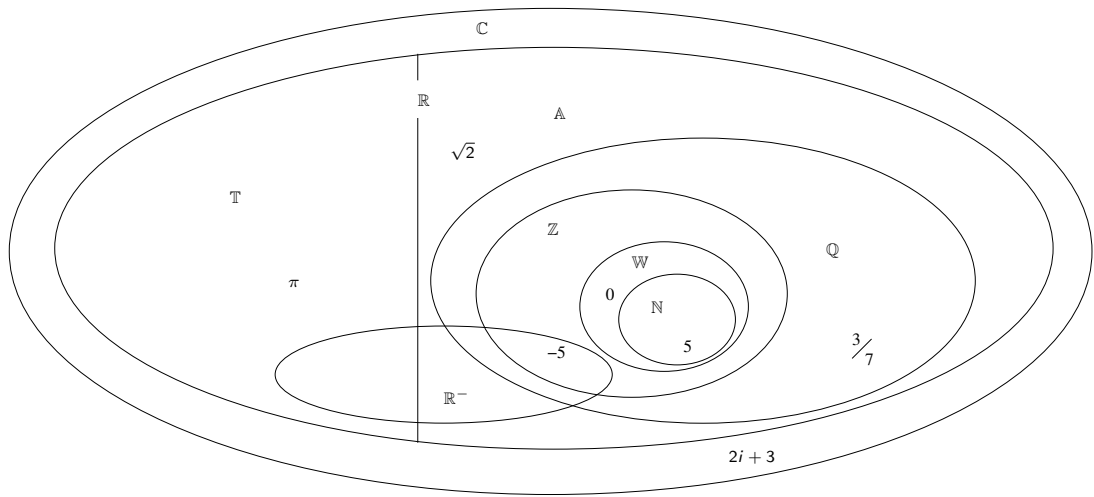


## Chapter 1 outline:

- ▶ Introduction, sets and elements (this past Monday)
- ▶ Set operations; visual verification of set propositions (Today)
- ▶ Introduction to SML; cardinality and Cartesian products (Friday)
- ▶ Making types and functions in SML (next week Wednesday)
- ▶ More about functions in SML; introduction to lists [Chapter 2] (next week Friday)

## Today:

- ▶ Set symbols and terminology
- ▶ Set notation
- ▶ Set operations
- ▶ Verifying set equivalence visually



5 is a natural number; *or* the collection of natural numbers contains 5.  $5 \in \mathbb{N}$

Adding 0 to the collection of natural numbers makes the collection of whole numbers.  $\mathbb{N} \cup \{0\} = \mathbb{W}$

Merging the algebraic numbers and the transcendental numbers makes the real numbers.  $\mathbb{A} \cup \mathbb{T} = \mathbb{R}$

Transcendental numbers are those real numbers which are not algebraic numbers.  $\mathbb{T} = \mathbb{R} - \mathbb{A}$

Nothing is both transcendental and algebraic, *or* the collection of things both transcendental and algebraic is empty.  $\mathbb{T} \cap \mathbb{A} = \emptyset$

Negative integers are both negative and integers.  $\mathbb{Z}^- = \mathbb{Z} \cap \mathbb{R}^-$

All integers are rational numbers.  $\mathbb{Z} \in \mathbb{R}$

Since all rational numbers are algebraic numbers and all algebraic numbers are real numbers, it follows that all rational numbers are real numbers.

$$\begin{aligned} \mathbb{Q} &\subseteq \mathbb{A} \\ \mathbb{A} &\subseteq \mathbb{R} \\ \therefore \mathbb{Q} &\subseteq \mathbb{R} \end{aligned}$$

## Axiom (Existence.)

*There is a set with no elements.*

## Axiom (Extensionality.)

*If every element of a set  $X$  is an element of a set  $Y$   
and every element of  $Y$  is an element of  $X$ , then  $X = Y$ .*

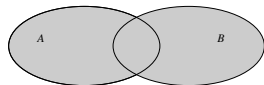
*Union*

$$A \cup B = \{ x \mid x \in A \text{ or } x \in B \}$$

$$\{1, 2, 3\} \cup \{2, 3, 4\} = \{1, 2, 3, 4\}$$

$$\{1, 2\} \cup \{3, 4\} = \{1, 2, 3, 4\}$$

$$\{1, 2\} \cup \{1, 2, 3\} = \{1, 2, 3\}$$



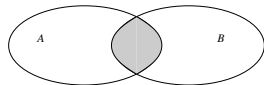
*Intersection*

$$A \cap B = \{ x \mid x \in A \text{ and } x \in B \}$$

$$\{1, 2, 3\} \cap \{2, 3, 4\} = \{2, 3\}$$

$$\{1, 2\} \cap \{3, 4\} = \emptyset$$

$$\{1, 2\} \cap \{1, 2, 3\} = \{1, 2\}$$



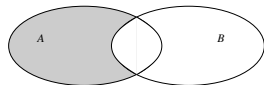
*Difference*

$$A - B = \{ x \mid x \in A \text{ and } x \notin B \}$$

$$\{1, 2, 3\} - \{2, 3, 4\} = \{1\}$$

$$\{1, 2\} - \{3, 4\} = \{1, 2\}$$

$$\{1, 2\} - \{1, 2, 3\} = \emptyset$$

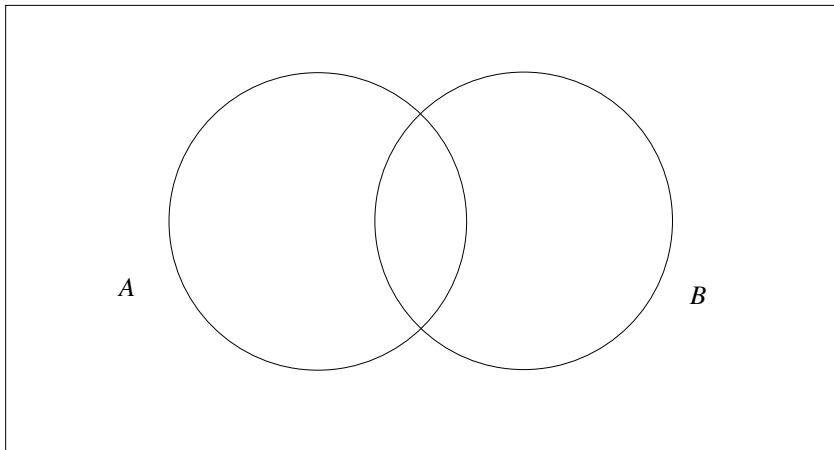


1.  $\{1, 2, 3, 4, 5\} \cup \{5, 6, 7\} =$

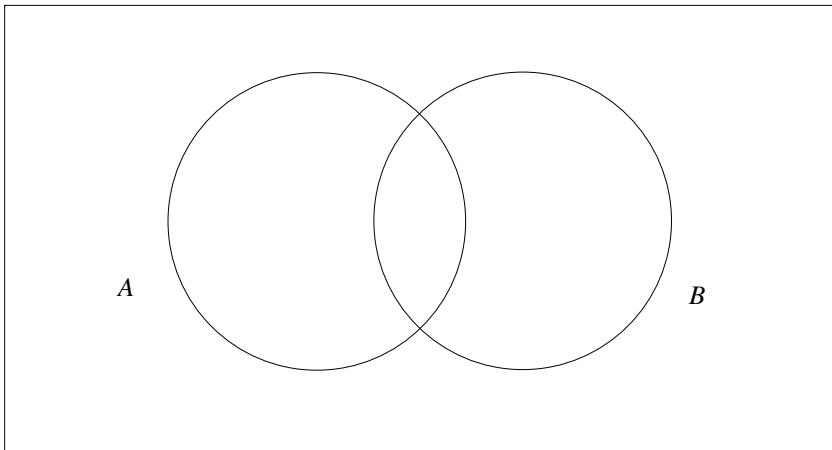
2.  $\{1, 2, 3, 4, 5\} \cap \{2, 4, 6, 8, 10\} =$

3.  $\{1, 2, 3, 4, 5\} - \{2, 3, 4\} =$

4.  $\{1, 2, 3, 4, 5\} - \{3, 4, 5, 6, 7\} =$

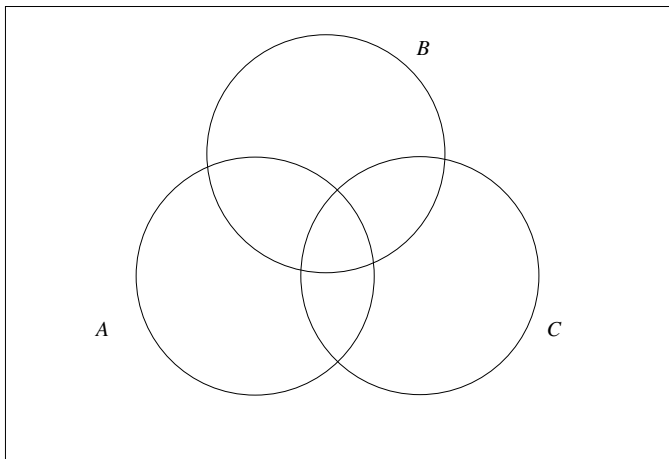


1.4.7.  $(A \cap B) - A$

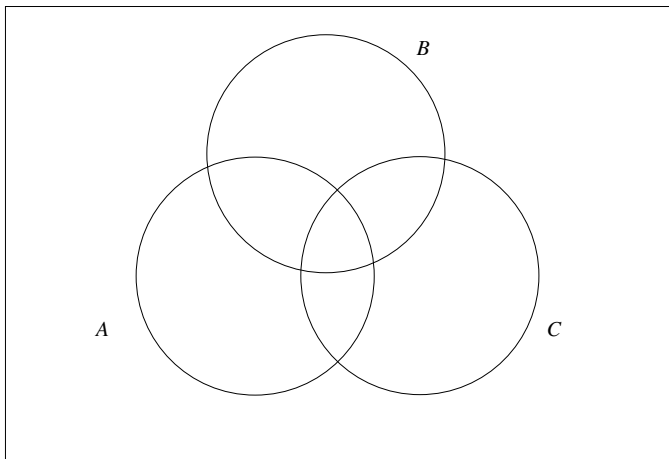


1.4.8.  $(A - B) \cup (B - A)$





1.4.9.  $(A \cup B) \cap (A \cup C)$



1.4.10.  $\overline{(A \cap B)} \cap (A \cup C)$

(Exercises 1.4.(11–18).)

$$-12 \in \mathbb{R}^-.$$

$$\mathbb{Q} \cap \mathbb{T} = \emptyset.$$

$$\mathbb{A} \subseteq \mathbb{C}.$$

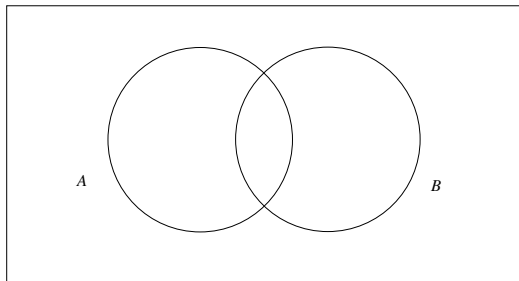
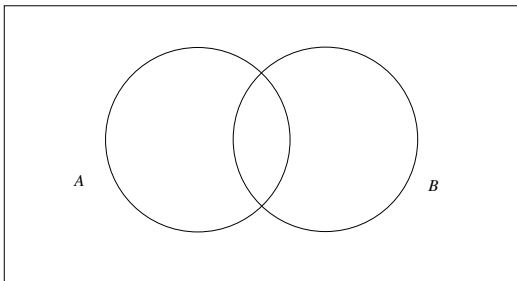
$$\frac{1}{63} \in \mathbb{Q} - \mathbb{R}.$$

$$\mathbb{R} \subseteq \mathbb{C} \cap \mathbb{R}^-$$

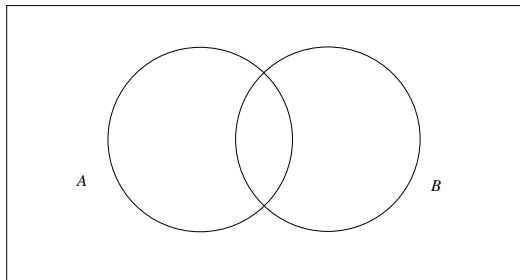
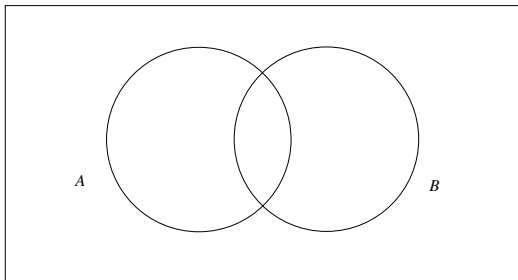
$$\mathbb{Z} - \mathbb{R}^- = \mathbb{W}.$$

$$4 \in \mathbb{C}.$$

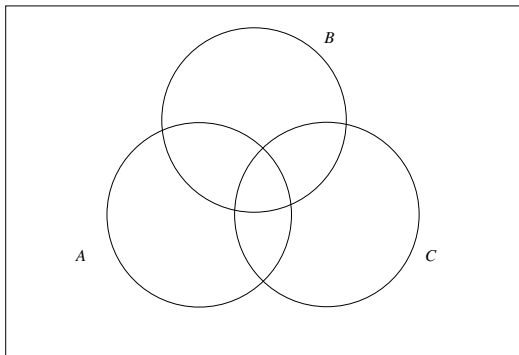
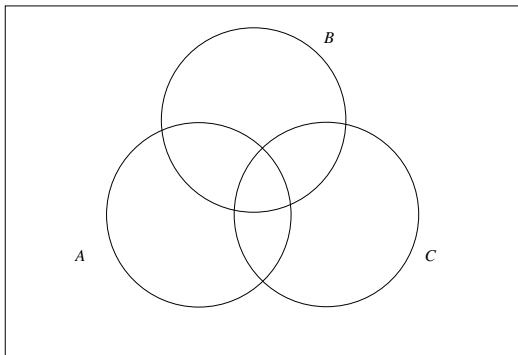
$$\mathbb{T} \cup \mathbb{Z} \subseteq \mathbb{A}.$$



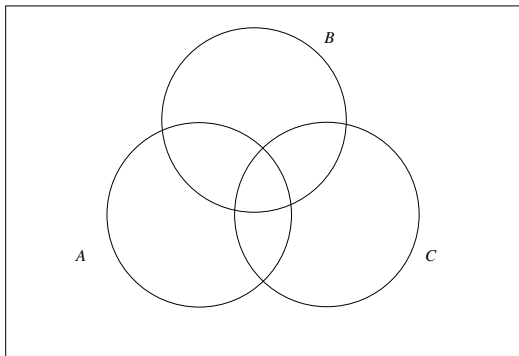
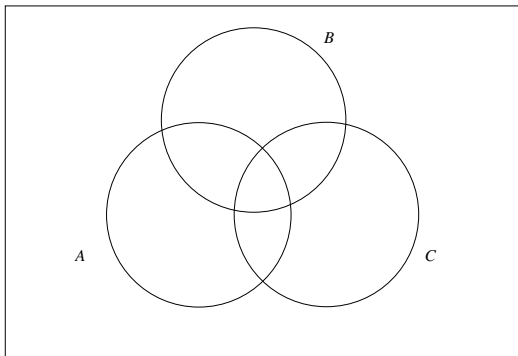
$$A \cup (A \cap B) = A$$



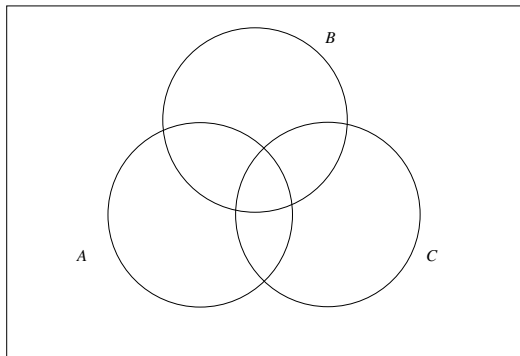
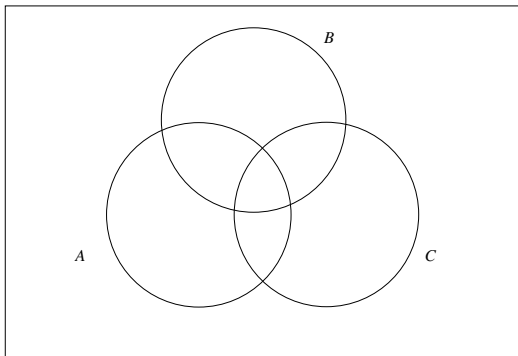
$$A \cup \bar{A} = \mathcal{U}$$



$$A \cup (B \cup C) = (A \cup B) \cup C$$

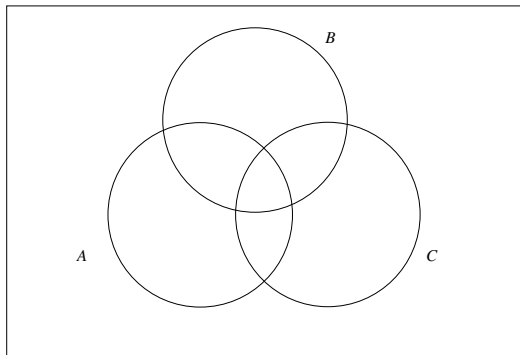
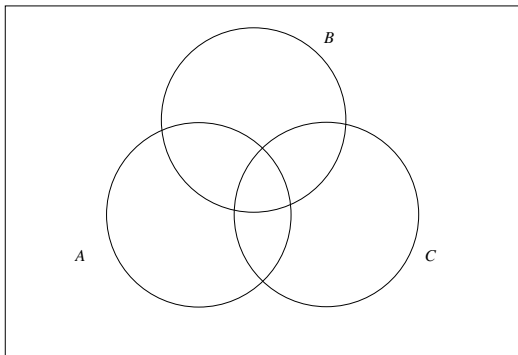


$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

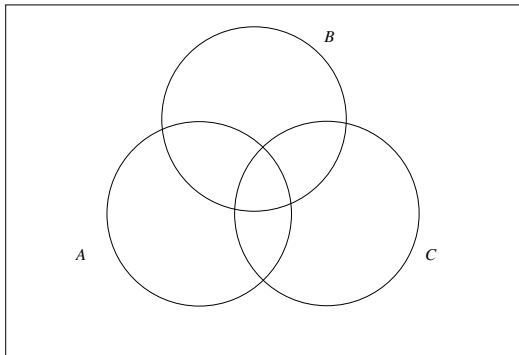
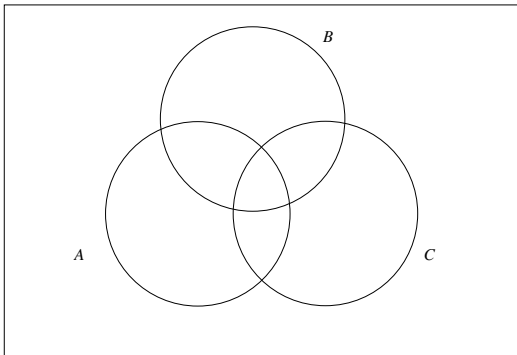


$$A \cap B = A - (A - B)$$

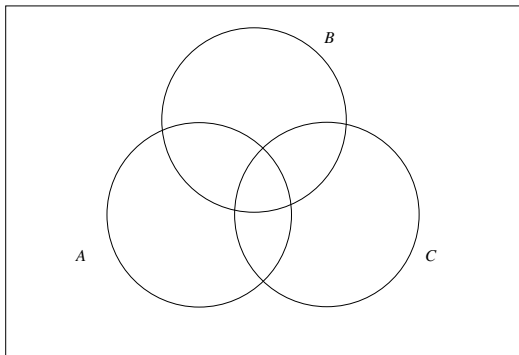
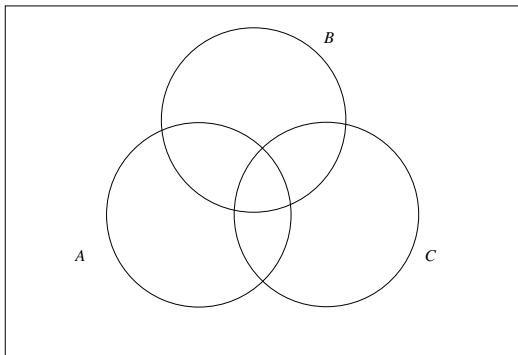




$$(A \cap C) - (C - B) = A \cap B \cap C$$



$$A \cup (A - B) = A$$



$$(A \cup (B - C)) \cap \bar{B} = A - B$$

**For next time:**

*Pg 12: 1.3.(11-14, 16)*

*Pg 16: 1.4.(1-6, 19)*

*Pg 20: 1.5.(8-11)*

*Remember, submit on paper after class or in the mesh box outside my office door (MEY 163).*

*Read 1.(6-9)*

*Take quiz*