

Chapter 4, Graphs:

- ▶ Concepts and implementation (Fri, Feb 11)
- ▶ Traversal (Mon, Feb 9)
- ▶ Minimum spanning trees (last week Wednesday and Friday)
- ▶ Single-source shortest paths (**Today**)

Today:

- ▶ (MST loose ends)
- ▶ The SSSP problem
- ▶ General concepts for SSSP algorithms
- ▶ The most unlucky graph for SSSP
- ▶ The Bellman-Ford algorithm plus analysis
- ▶ Dijkstra's algorithm plus analysis

Minimum Spanning Tree Problem

Given a weighted, undirected graph, find the tree with least-total weight that connects all the vertices, if one exists.

- ▶ Both are defined for weighted graphs
- ▶ Both produce trees as a result
- ▶ Both minimize by weight
- ▶ For each we have two algorithms

Input is only a graph

Problem usually is described on an undirected graph

Goal is to minimize total weight

There is no clear winner between the algorithms

Single-Source Shortest Paths Problem

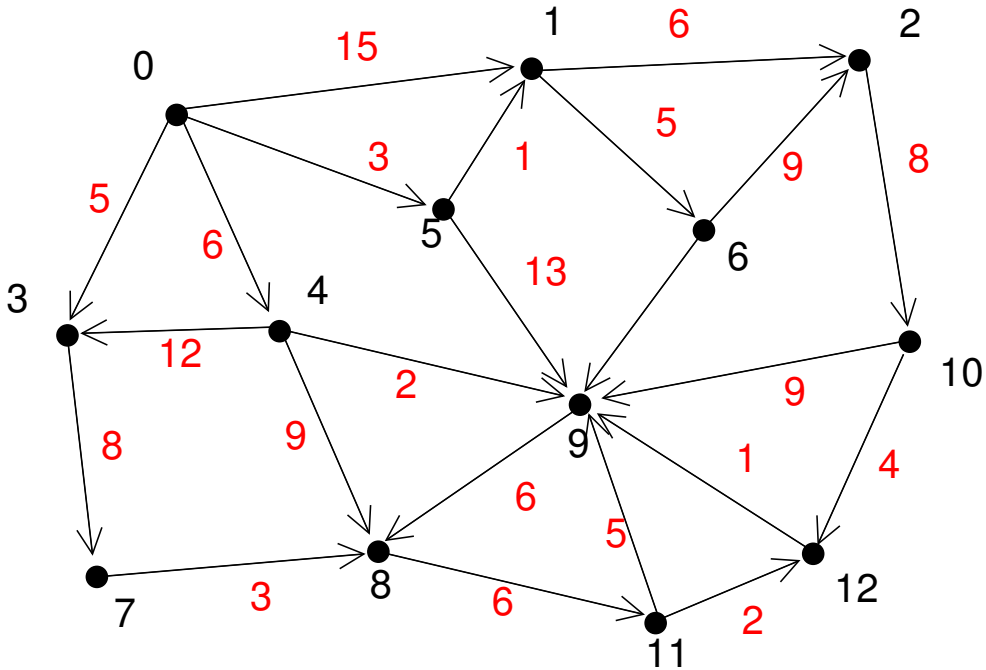
Given a weighted directed graph and a source vertex, find the tree comprising the shortest paths from that source to all other reachable vertices.

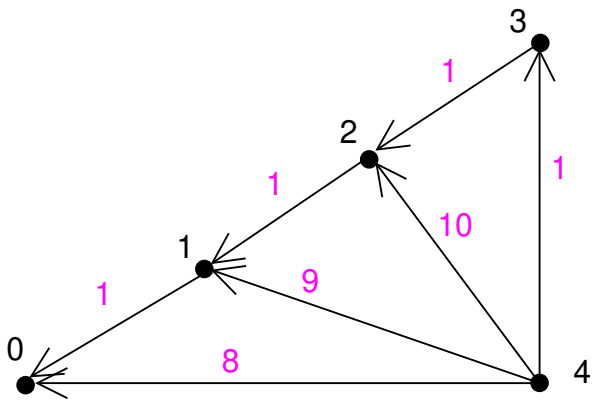
Input is a graph and a starting point

Problem usually is described on a directed graph

Goal is to minimize weight on each path

One algorithm is clearly more efficient





Let X be the set of vertices whose distance bounds are correct, that is, $v \in X$ if $\text{distances}[v]$ is the total weight of the shortest path from s to v . For a single-source shortest path algorithm to be correct, all vertices reachable from s are in set X at termination, and if *all* vertices are reachable, this implies $X = V$. Let Y be the set of vertices that have been removed from the priority queue. Our intent is that $Y \subseteq X$: all vertices have correct distance bound at the time they are removed from the priority queue, though at any point there may also be some correct ones still in the priority queue. We claim

Invariant (Main loop of Dijkstra's algorithm)

Let X and Y be as defined above.

- (a) $Y \subseteq X$.
- (b) If v is the vertex in the priority queue with least distance bound, then $v \in X$.
- (c) $|Y|$ is the number of iterations completed.

Coming up:

Do **MST** project (suggested by Monday, Feb 28)

Due **Tues, Mar 1** [end of day]

Read Section 4.5

Do Exercises 4.(47, 48, 56) (note correction on 4.52)

Take SSSP quiz

Do **SSSP** project (suggested by Thursday, Mar 3)

Due **Fri, Mar 4** [class time]

Read Sections 5.(1 & 2)

Do Exercises 5.(2 & 6)

Take BST quiz