

Chapter 3 roadmap:

- ▶ Propositions, boolean logic, logical equivalences. **Game 1** (last week Friday)
- ▶ Conditional propositions. **SML** (Monday)
- ▶ Arguments. **Game 2** (Today)
- ▶ Predicates and quantification. **SML** (Friday)
- ▶ Quantified arguments. **Game 3** (next week Monday)
- ▶ Review for test (next week Wednesday)

Today:

- ▶ Define arguments
- ▶ Consider known “syllogisms”
- ▶ Practice verifying argument forms (Game 2)

$$((q \wedge (p \wedge (p \vee q))) \vee (q \wedge \sim p)) \wedge \sim q$$

$$\equiv ((q \wedge p) \vee (q \wedge \sim p)) \wedge \sim q \quad \text{Absorption}$$

$$\equiv (q \wedge (p \vee \sim p)) \wedge \sim q \quad \text{Distributivity}$$

$$\equiv (q \wedge T) \wedge \sim q \quad \text{Negation}$$

$$\equiv q \wedge \sim q \quad \text{Identity}$$

$$\equiv F \quad \text{Negation}$$

WRONG!

$$((q \wedge (p \wedge (p \vee q))) \vee (q \wedge \sim p)) \wedge \sim q$$

$$\equiv ((q \wedge p) \vee (q \wedge \sim p)) \wedge \sim q \quad \text{Absorption}$$

$$\equiv (q \wedge p) \vee ((q \wedge \sim p) \wedge \sim q) \quad \text{Associativity}$$

Valid argument

If it is Monday, then it is raining
It is Monday.
Therefore it is raining.

$p \rightarrow q$

p

$\therefore q$

p	q	$p \rightarrow q$	q
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	F

Annotations:
- Above the first column: p (premise)
- Above the second column: q (premise)
- Above the third column: $p \rightarrow q$ (premise)
- Above the fourth column: q (conclusion)
- An arrow labeled "critical row" points to the second row (where p is true and q is false).

Invalid argument

If it is raining, then there are clouds
There are clouds.
Therefore it is raining.

$p \rightarrow q$

q

$\therefore p$

p	q	$p \rightarrow q$	p
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	F

Diagram labels: *premise* (above q), *premise* (above $p \rightarrow q$), *conclusion* (above p).
Critical rows: T, F and F, T (indicated by arrows from the right).

Alternate definition of validity

Valid argument

p	q	$p \rightarrow q$	$(p \wedge (p \rightarrow q)) \rightarrow q$
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	T

Invalid argument

p	q	$p \rightarrow q$	$(q \wedge (p \rightarrow q)) \rightarrow p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

Modus tollens

If it is spring, then the daffodils bloom.
The daffodils aren't blooming.
Therefore it is not spring.

p	q	$p \rightarrow q$	$\sim q$	$\sim p$
T	T	T	F	
T	F	F	T	
F	T	T	F	
F	F	T	T	T

Modus Ponens

$$p \rightarrow q$$

$$p$$

$$\therefore q$$

Modus Tollens

$$p \rightarrow q$$

$$\sim q$$

$$\therefore \sim p$$

Generalization

$$p$$

$$\therefore p \vee q$$

Specialization

$$p \wedge q$$

$$\therefore p$$

Elimination

$$p \vee q$$

$$\sim p$$

$$\therefore q$$

Transitivity

$$p \rightarrow q$$

$$q \rightarrow r$$

$$\therefore p \rightarrow r$$

Division into cases

$$p \vee q$$

$$p \rightarrow r$$

$$q \rightarrow r$$

$$\therefore r$$

Contradiction

$$p \rightarrow F$$

$$\therefore \sim p$$

Proof by contradiction

$$p \rightarrow F$$
$$\therefore \sim p$$

p	$p \rightarrow F$	$\sim p$
T	F	F
F	T	T

\leftarrow *critical row*

Mod Pon	Mod Tol	Generalization	Specialization	Elimination	Transitivity	Div into cases	Contradiction
$p \rightarrow q$	$p \rightarrow q$	p	$p \wedge q$	$p \vee q$	$p \rightarrow q$	$p \vee q$	$p \rightarrow F$
p	$\sim q$	$\therefore p \vee q$	$\therefore p$	$\sim p$	$q \rightarrow r$	$p \rightarrow r$	$\therefore \sim p$
$\therefore q$	$\therefore \sim p$			$\therefore q$	$\therefore p \rightarrow r$	$q \rightarrow r$	
						$\therefore r$	

3.9.1

- (a) $t \rightarrow u$
- (b) $p \vee \sim q$
- (c) $p \rightarrow (u \rightarrow r)$
- (d) q
- (e) $\therefore t \rightarrow r$

Mod Pon	Mod Tol	Generalization	Specialization	Elimination	Transitivity	Div into cases	Contradiction
$p \rightarrow q$	$p \rightarrow q$	p	$p \wedge q$	$p \vee q$	$p \rightarrow q$	$p \vee q$	$p \rightarrow F$
p	$\sim q$	$\therefore p \vee q$	$\therefore p$	$\sim p$	$q \rightarrow r$	$p \rightarrow r$	$\therefore \sim p$
$\therefore q$	$\therefore \sim p$			$\therefore q$	$\therefore p \rightarrow r$	$q \rightarrow r$	
						$\therefore r$	

3.9.2

(a) $p \rightarrow t$

(b) $\sim (q \rightarrow t) \rightarrow w$

(c) $p \vee q$

(d) $\sim w$

(e) $\therefore t$

Mod Pon	Mod Tol	Generalization	Specialization	Elimination	Transitivity	Div into cases	Contradiction
$p \rightarrow q$	$p \rightarrow q$	p	$p \wedge q$	$p \vee q$	$p \rightarrow q$	$p \vee q$	$p \rightarrow F$
p	$\sim q$	$\therefore p \vee q$	$\therefore p$	$\sim p$	$q \rightarrow r$	$p \rightarrow r$	$\therefore \sim p$
$\therefore q$	$\therefore \sim p$			$\therefore q$	$\therefore p \rightarrow r$	$q \rightarrow r$	
						$\therefore r$	

3.9.8

- (a) w
- (b) $q \rightarrow r$
- (c) $t \rightarrow s$
- (d) $u \rightarrow s$
- (e) $(\sim t \wedge \sim u) \rightarrow \sim w$
- (f) $(s \vee y) \rightarrow (p \rightarrow q)$
- (g) $\sim (p \rightarrow r) \vee x$
- (h) $\therefore x$

Mod Pon	Mod Tol	Generalization	Specialization	Elimination	Transitivity	Div into cases	Contradiction
$p \rightarrow q$	$p \rightarrow q$	p	$p \wedge q$	$p \vee q$	$p \rightarrow q$	$p \vee q$	$p \rightarrow F$
p	$\sim q$	$\therefore p \vee q$	$\therefore p$	$\sim p$	$q \rightarrow r$	$p \rightarrow r$	$\therefore \sim p$
$\therefore q$	$\therefore \sim p$			$\therefore q$	$\therefore p \rightarrow r$	$q \rightarrow r$	
						$\therefore r$	

3.9.9

(a) $p \rightarrow q$

(b) x

(c) $\sim (p \vee w) \rightarrow r$

(d) $q \rightarrow u$

(e) $x \rightarrow t$

(f) $w \rightarrow u$

(g) $r \vee s$

(h) $r \rightarrow F$

(i) $\therefore t \wedge s \wedge u$

Mod Pon	Mod Tol	Generalization	Specialization	Elimination	Transitivity	Div into cases	Contradiction
$p \rightarrow q$	$p \rightarrow q$	p	$p \wedge q$	$p \vee q$	$p \rightarrow q$	$p \vee q$	$p \rightarrow F$
p	$\sim q$	$\therefore p \vee q$	$\therefore p$	$\sim p$	$q \rightarrow r$	$p \rightarrow r$	$\therefore \sim p$
$\therefore q$	$\therefore \sim p$			$\therefore q$	$\therefore p \rightarrow r$	$q \rightarrow r$	
						$\therefore r$	

3.9.10

(a) $u \rightarrow \sim p$

(b) $(\sim p \vee q) \rightarrow (r \rightarrow s)$

(c) $u \wedge \sim w$

(d) $t \rightarrow s$

(e) $(\sim t \wedge \sim r) \rightarrow w$

(f) $\therefore s$

For next time:

Pg 119: 3.8.(3 & 5)

Pg 122: 3.9.(3-7)

Read carefully 3.(10 & 11)

Skim 3.(12 & 13)

Take quiz