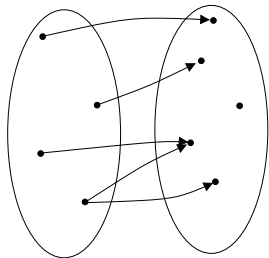


Chapter 7 outline:

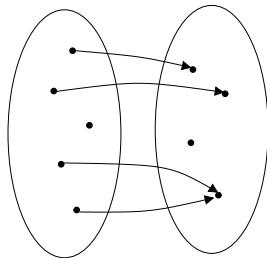
- ▶ Introduction, function equality, and anonymous functions (last week Monday)
- ▶ Image and inverse images (last week Wednesday)
- ▶ Function properties, composition, and applications to programming (Monday)
- ▶ Function composition (**Today, i.e. Wednesday**)
- ▶ Cardinality (**Today, i.e. Friday**)
- ▶ *Practice quiz* and Countability (next week Monday)
- ▶ Review (next week Wednesday)
- ▶ Test 3, on Ch 6 & 7 (next week Friday)

Today:

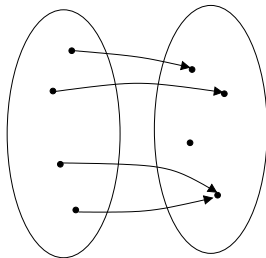
- ▶ Formal definition of cardinality
- ▶ If $A \cap B = \emptyset$, then $|A \cup B| = |A| + |B|$
- ▶ If $f : A \rightarrow B$ is one-to-one, then $|A| \leq |B|$.



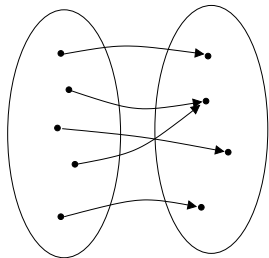
Not a function



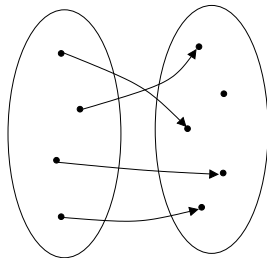
Not a function



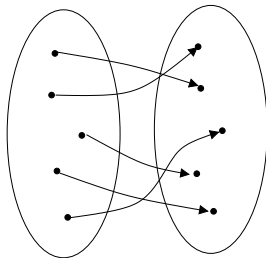
A function but not
one-to-one or onto



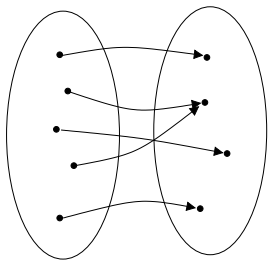
Onto, not one-to-one



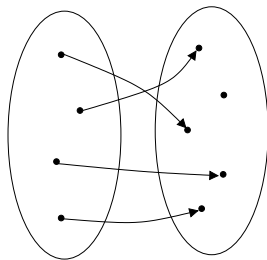
One-to-one, not onto



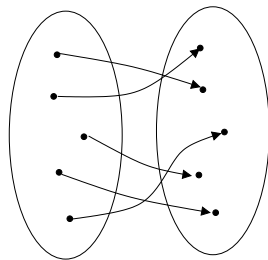
One-to-one correspondence



Onto, not one-to-one
 $|X| \geq |Y|$



One-to-one, not onto
 $|X| \leq |Y|$



One-to-one correspondence
 $|X| = |Y|$

Two finite sets X and Y have the *the same cardinality* as each other if there exists a one-to-one correspondence from X to Y .

To use this *analytically*:

Suppose X and Y have the same cardinality. Then let f be a one-to-one correspondence from X to Y .

f is both onto and one-to-one.

To use this *synthetically*:

Given sets X and Y ...

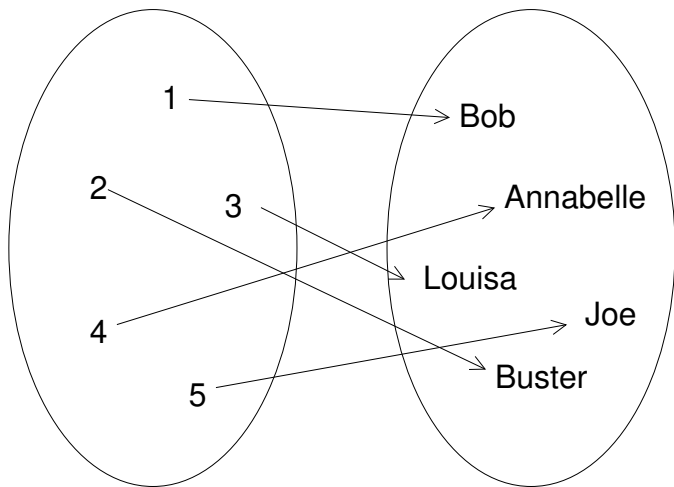
[Define f] Let $f : X \rightarrow Y$ be a function defined as ...

Suppose $y \in Y$. *Somehow find $x \in X$ such that $f(x) = y$.* Hence f is onto.

Suppose $x_1, x_2 \in X$ such that $f(x_1) = f(x_2)$. *Somehow show $x_1 = x_2$.* Hence f is one-to-one.

Since f is a one-to-one correspondence, X and Y have the same cardinality as each other.

A finite set X has cardinality $n \in \mathbb{N}$, which we write as $|X| = n$, if there exists a one-to-one correspondence from $\{1, 2, \dots, n\}$ to X . Moreover, $|\emptyset| = 0$.

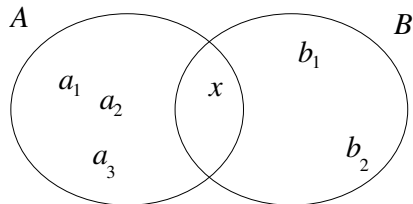


Theorem 7.12. If A and B are finite, disjoint sets, then $|A \cup B| = |A| + |B|$.

Theorem 7.13. If A and B are finite sets and $f : A \rightarrow B$ is one-to-one, then $|A| \leq |B|$.

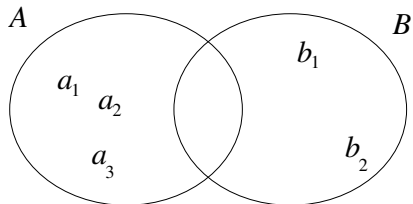
Exercise 7.9.5. If A and B are finite sets and $f : A \rightarrow B$ is onto, then $|A| \geq |B|$.

$$A \cap B = \emptyset \rightarrow |A \cup B| = |A| + |B|$$



$$|A \cup B| = |\{a_1, a_2, a_3, x, b_1, b_2\}| = 6$$

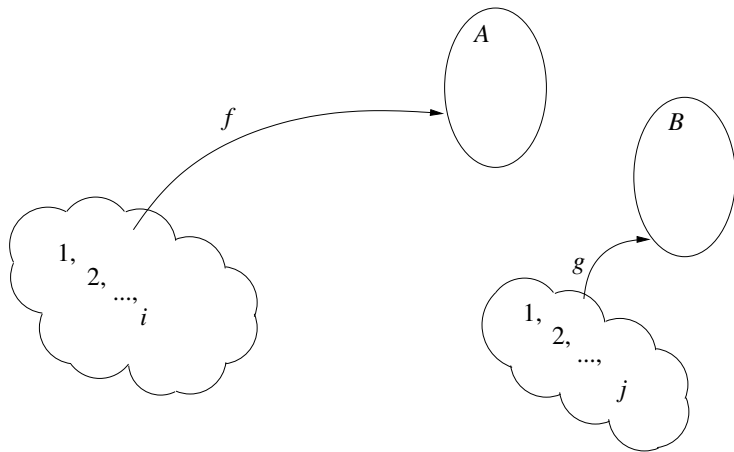
$$\begin{aligned} |A| + |B| &= \\ &= |\{a_1, a_2, a_3, x\}| + |\{x, b_1, b_2\}| \\ &= 4 + 3 = 7 \end{aligned}$$



$$|A \cup B| = |\{a_1, a_2, a_3, b_1, b_2\}| = 5$$

$$\begin{aligned} |A| + |B| &= \\ &= |\{a_1, a_2, a_3\}| + |\{b_1, b_2\}| \\ &= 3 + 2 = 5 \end{aligned}$$

$$A \cap B = \emptyset \rightarrow |A \cup B| = |A| + |B|$$



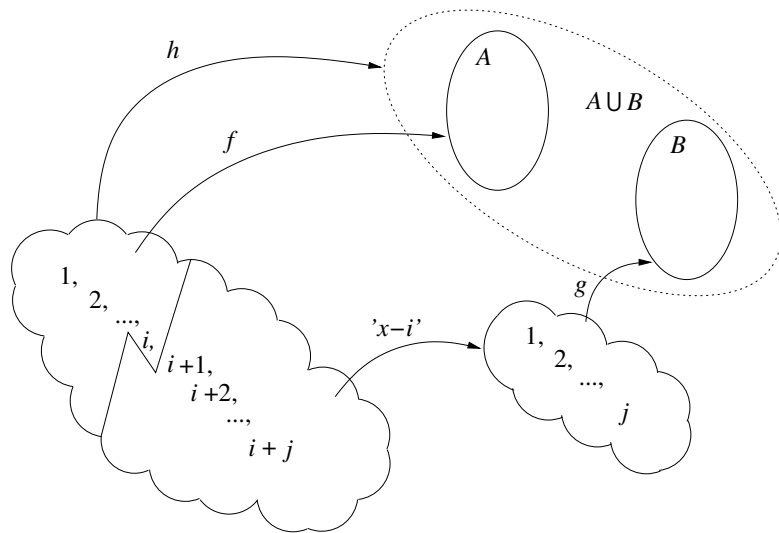
$$A \cap B = \emptyset \rightarrow |A \cup B| = |A| + |B|$$

x	f
1	Zed
2	Yelemis
3	Xavier

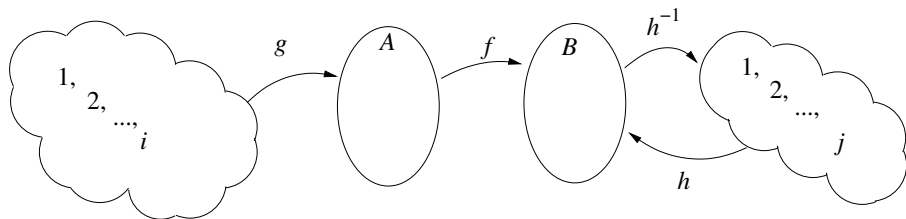
x	g
1	Wilhelmina
2	Valerie
3	Ursula
4	Tassie

x	h
1	$f(1) =$ Zed
2	$f(2) =$ Yelemis
3	$f(3) =$ Xavier
4	$g(4 - 3) = g(1) =$ Wilhelmina
5	$g(5 - 3) = g(2) =$ Valerie
6	$g(6 - 3) = g(3) =$ Ursula
7	$g(7 - 3) = g(4) =$ Tassie

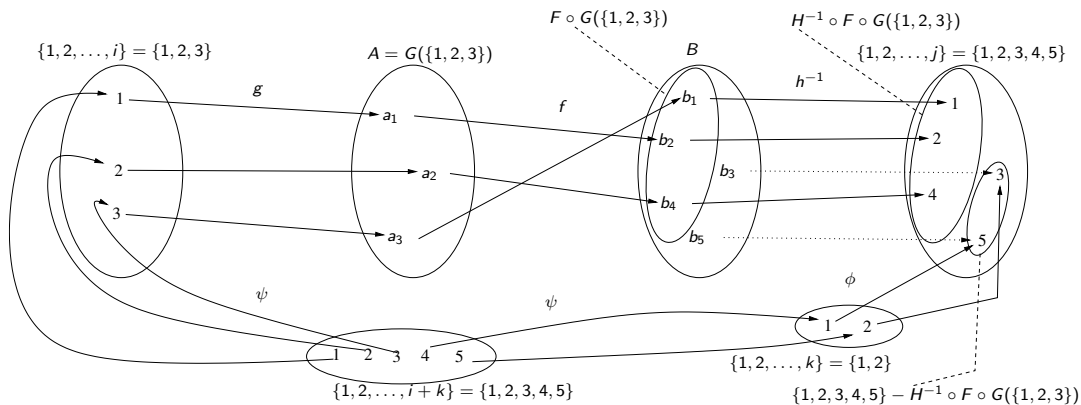
$$A \cap B = \emptyset \rightarrow |A \cup B| = |A| + |B|$$



$f : A \rightarrow B$ is one-to-one $\rightarrow |A| \leq |B|$



$f : A \rightarrow B$ is one-to-one $\rightarrow |A| \leq |B|$



For next time:

Pg 359: 7.9.(1 & 2)