

Chapter 4 roadmap:

- ▶ Subset proofs (Monday)
- ▶ Set equality and emptiness proofs (Wednesday)
- ▶ Conditional and biconditional proofs (**Today**)
- ▶ Proofs about powersets (new week Wednesday)
- ▶ From theorems to algorithms (next week Friday)
- ▶ (Start Chapter 5 week after next)

Today:

- ▶ Proofs of conditional propositions
- ▶ Proofs about numbers
- ▶ Proofs of biconditional propositions

General forms:

1. Facts (p)

Set forms

1. Subset $X \subseteq Y$
2. Set equality $X = Y$
3. Set emptiness $X = \emptyset$

2. Conditionals ($p \rightarrow q$)

3. Biconditionals ($p \leftrightarrow q$)

Hypothetical conditional from **Game 3**:

To prove $p \rightarrow q$

Suppose p

...

q

$p \rightarrow q$

An integer x is *even* if $\exists k \in \mathbb{Z} \mid x = 2k$.

An integer x is *odd* if $\exists k \in \mathbb{Z} \mid x = 2k + 1$.

“Axiom 3.” If $x, y \in \mathbb{Z}$, then $x + y \in \mathbb{Z}$. (*Closure of addition*)

“Axiom 4.” If $x, y \in \mathbb{Z}$, then $x \cdot y \in \mathbb{Z}$. (*Closure of multiplication*)

“Axiom 5.” If $x \in \mathbb{Z}$, then x is even iff x is not odd.

$\forall x, y \in \mathbb{Z}$, $x \mid y$ (read, “ x divides y ”) if $\exists k \in \mathbb{Z} \mid x \cdot k = y$.

Note that $y/x = k$ or $\frac{y}{x} = k$ or $x \overline{\mid} \frac{k}{y}$.

For next time:

Pg 162: 4.5.(1, 4, 5)

Pg 164: 4.6.(2 & 5)

Pg 165: 4.7.(1 & 6)

Review 2.4, especially Ex 2.4.15

Skim 4.9

Take quiz on Schoology