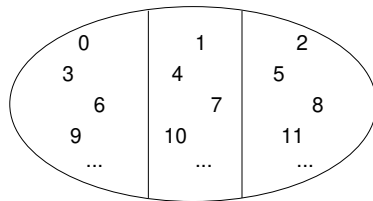
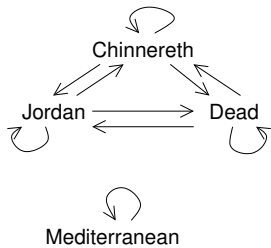
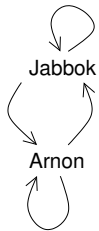


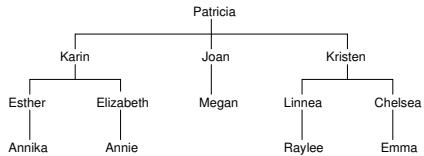
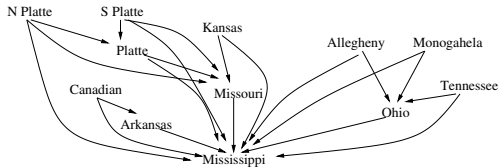
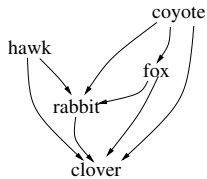
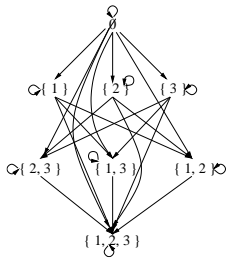
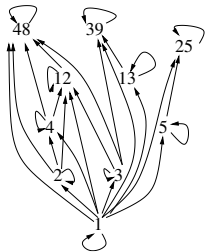
Chapter 5 roadmap:

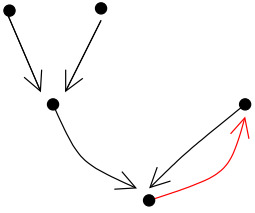
- ▶ Introduction to relations (Monday, Feb 27)
- ▶ Properties of relations (Wednesday, Mar 1 and Friday, Mar 3)
- ▶ Transitive closure (Monday)
- ▶ Partial order relations (**Today**)
- ▶ Review for Test 2 (Friday)
- ▶ Test 2 on Chapters 4 & 5 (next week Monday)

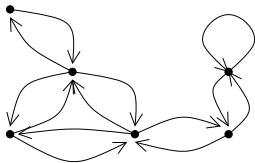
Today:

- ▶ Antisymmetry
- ▶ Partial order relations
- ▶ Topological sort



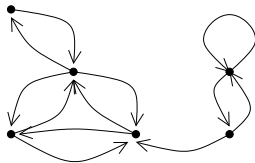






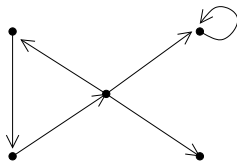
symmetric

All arrows
have a back arrow.



asymmetric
(not symmetric)

There exists an arrow
without a back arrow.



antisymmetric
("very" not symmetric)

No arrows have back arrows
except self loops.

Formal definition:

A relation R on a set X is antisymmetric if $\forall x, y \in X$, if $(x, y) \in R$ and $(y, x) \in R$, then $x = y$.

Informal definition:

If both an arrow and its reverse exist in an antisymmetric relation R , then that arrow must be a self loop (and, hence, it is its own reverse).

Alternate formal definition:

A relation R on a set X is antisymmetric if $\forall (x, y) \in R$, either $x = y$ or $(y, x) \notin R$.

Rock beats scissors; scissors beats paper; paper beats rock.

Grasshopper eats corn; mouse eats corn; mouse eats grasshopper; snake eats mouse; hawk eats mouse; hawk eats snake.

Aurelia is better than Gwendolyn at pitching; Gwendolyn is better than Aurelia at batting.

Peter Pan is shorter than Treasure Island; Treasure Island is shorter than Anna Karenina; Anna Karenina is shorter than The Count of Monte Christo.

CSCI 235 is a prereq for CSCI 245; CSCI 245 is a prereq for CSCI 345; CSCI 243 is a prereq for CSCI 345; MATH 231 is a prereq for MATH 245; CSCI 345 is a prereq for CSCI 381; MATH 245 is a prereq for CSCI 381.

I married a widow with a grown daughter; my father, a widower, then married my step-daughter. Thus I am my own step-grampa. (The relation in this example is "is biological ancestor of or step-ancestor of".)

A relation R on a set X is antisymmetric if $\forall x, y \in X$, if $(x, y) \in R$ and $(y, x) \in R$, then $x = y$.

Ex 5.8.9. Prove that $|$ (divides) on \mathbb{N} is antisymmetric.

Proof. Suppose $x, y \in \mathbb{N}$, $x|y$, and $y|x$ (that is, $(x, y), (y, x) \in |$). By definition of divides, there exists $i, j \in \mathbb{N}$ such that

$$\begin{aligned}x &= i \cdot y \\y &= j \cdot x\end{aligned}$$

Then

$$\begin{aligned}x &= i \cdot j \cdot x && \text{by substitution} \\1 &= i \cdot j && \text{by cancellation} \\i &= j = 1 && \text{by arithmetic} \\x &= y && \text{by identity}\end{aligned}$$

Therefore $|$ is antisymmetric by definition. \square

Antisymmetry:

A relation R on a set X is *antisymmetric* if $\forall x, y \in X$, if $(x, y) \in R$ and $(y, x) \in R$, then $x = y$.

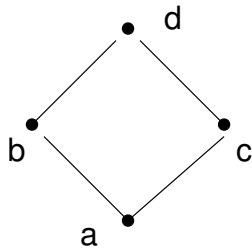
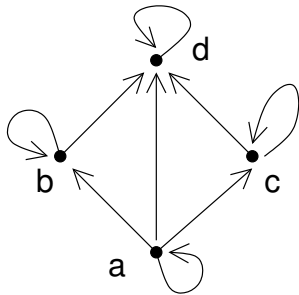
Partial order relation:

A *partial order relation* (or just *partial order*) is a relation that is reflexive, transitive, and antisymmetric.

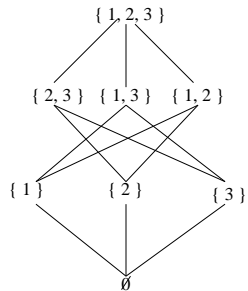
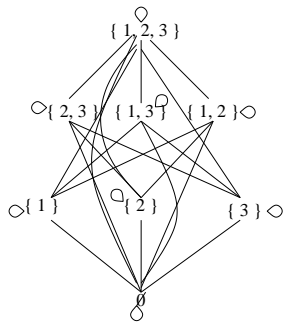
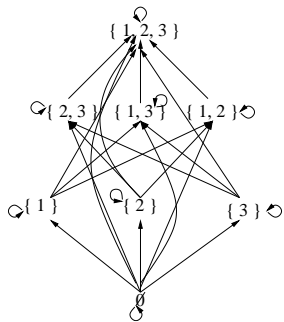
A *strict partial order (relation)* is a relation that is irreflexive, transitive and antisymmetric.

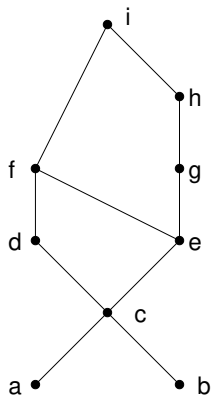
Partially ordered set:

A *partially ordered set* or *poset* is a set together with a partial order on that set.



$$R = \{(a, a), (a, b), (a, c), (a, d), (b, b), (b, d), (c, c), (c, d), (d, d)\}$$





Comparable: $a \preceq c, d \preceq f, e \preceq f, e \preceq h, c \preceq i$

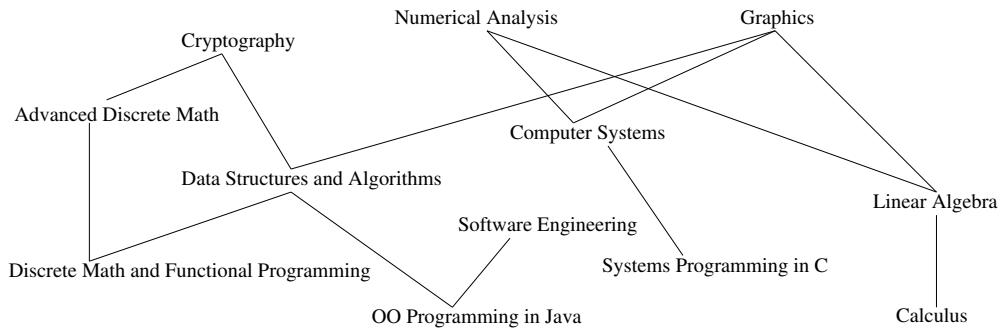
Not comparable: a and b ; d and e ; f and h

Maximal and greatest: i

Minimal: a and b

No least

Everyday examples: Preparing a meal, writing a term paper, getting dressed



A partial order R on a set X is a *total order* if for all $x, y \in X$, either $x \preceq y$ or $y \preceq x$, that is, x and y are comparable.

Standard example of a total order: \leq .

A *partial order relation* (or just *partial order*) is a relation that is reflexive, transitive, and antisymmetric.

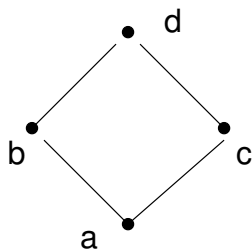
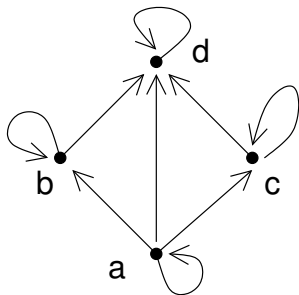
A partial order R on a set X is a *total order* if for all $x, y \in X$, either $x \preceq y$ or $y \preceq x$, that is, x and y are comparable.

A *topological sort* of a partial order R is a total order that is a superset of R .

| (divides) \leq

is prerequisite for Ralph takes before

can put on before you put on before



$$R = \{(a, a), (a, b), (a, c), (a, d), (b, b), (b, d), (c, c), (c, d), (d, d)\}$$

A topological sort for R : $R \cup \{(b, c)\}$, written as a, b, c, d

Another topological sort for R : $R \cup \{(c, b)\}$, written as a, c, b, d

For next time:

Pg 226: 5.8.(1-5)

Pg 231 5.9.(1 & 8)

For Wednesday, Mar 22:

Read 6.(1-3)

Take quiz