Where we are:

- Making types in SML (last week Wednesday)
- Functions in SML (last week Friday)
- Lists and functions on lists (Wednesday)
- Powersets; a language processor (**Today**)
- Propositional forms, logical equivalence [Start Chapter 3] (Friday)

Today:

- Powersets
  - Definition
  - Exploration
- A language processor
  - Case expressions and option types
  - The language processor itself
  - Introducing the semester project

Review

- List literals: `[1, 4, 12, 3]`, `[]`
- Analytic operations: `hd`, `tl`
- Synthetic operations: `::` (cons), `@` (cat)
- Lists vs tuples
- Type analysis problems :

$$\underbrace{[\, \underbrace{\underbrace{\texttt{tl}(\underbrace{[\underbrace{5}_{\text{int}}, \underbrace{12}_{\text{int}}, \underbrace{6}_{\text{int}}]}_{\text{int list}})}_{\text{int list}} \;\texttt{@}\; \underbrace{[\underbrace{8}_{\text{int}}, \underbrace{9}_{\text{int}}]}_{\text{int list}}}_{\text{int list}}\,]}_{\text{int list list}}$$

- Lists as models for sets

Powersets

- Informal definition: The powerset of a set is the set of all subsets of that set.
- Formal definition: The powerset of a set X is

$$\mathscr{P}(X) = \{\ Y \mid Y \subseteq X\}$$

- For "set of sets," think "box of boxes."
- Examples:

Why powersets seem to throw some people:

- The elements of a powerset are themselves sets.
- Suppose $X \subseteq \mathcal{U}$. Then
  - If $x \in X$, then $x \in \mathcal{U}$
  - $\mathscr{P}(X) \not\subseteq \mathcal{U}$, but rather $\mathscr{P}(X) \subseteq \mathscr{P}(\mathcal{U})$
  - If $A \in \mathscr{P}(X)$, then $A \in \mathscr{P}(\mathcal{U})$
- $\mathscr{P}(\emptyset) = \{\emptyset\} \neq \emptyset$. $|\emptyset| = 0$, but $|\{\emptyset\}| = 1$

Which are true?

$\{3\} \in \mathscr{P}(\{1, 2, 3, 4, 5\})$

$3 \in \mathscr{P}(\{1, 2, 3, 4, 5\})$

$\{3\} \subseteq \mathscr{P}(\{1, 2, 3, 4, 5\})$

$3 \subseteq \mathscr{P}(\{1, 2, 3, 4, 5\})$

$a \in A$ iff $\{a\} \in \mathscr{P}(A)$

$a \in A$ iff $\{a\} \subseteq \mathscr{P}(A)$

$A \subseteq B$ iff $A \subseteq \mathscr{P}(B)$

$A \subseteq B$ iff $A \in \mathscr{P}(B)$

Which are true?

$A \subseteq B$ iff $A \subseteq \mathscr{P}(B)$

$\{A\} \subseteq \mathscr{P}(A)$

$A \in \mathscr{P}(A)$

$\{A\} \in \mathscr{P}(A)$

$\mathbb{Z} \in \mathscr{P}(\mathbb{R})$

$\emptyset = \mathscr{P}(\emptyset)$

Note that

- $a \in A$ iff $\{a\} \in \mathscr{P}(A)$
- $A \subseteq B$ iff $A \in \mathscr{P}(B)$
- $A \subseteq B$ iff $\mathscr{P}(A) \subseteq \mathscr{P}(B)$
- $\mathscr{P}(\emptyset) = \{\emptyset\} \neq \emptyset$

Observe

$$\mathscr{P}(\{1,2,3\}) = \{\ \emptyset,\ \{1\},\{2\},\{3\},\ \{1,2\},\{1,3\},\{2,3\},\ \{1,2,3\}\ \} = \{\ \{1\},\{1,2\},\{1,3\},\{1,2,3\},\ \emptyset,\{2\},\{3\},\{2,3\}\ \}$$

$$= \mathscr{P}(\{2,3\}) \cup \begin{bmatrix}\text{1 added to each set} \\ \text{of } \mathscr{P}(\{2,3\})\end{bmatrix} = \mathscr{P}(\{2,3\}) \cup \{\ \{1\} \cup X \mid X \in \mathscr{P}(\{2,3\})\ \}$$

If $a \in A$, then $\mathscr{P}(A) = \mathscr{P}(A - \{a\}) \cup \{\ \{a\} \cup X \mid X \in \mathscr{P}(A - \{a\})\ \}$

What is $|\mathscr{P}(X)|$ in terms of $|X|$?

Grammar:

$$
\begin{aligned}
\textit{Sentence} &\rightarrow \textit{NounPhrase Predicate PrepPhrase}_{opt} \\
\textit{NounPhrase} &\rightarrow \textit{Article Adjective}_{opt}\ \textit{Noun} \\
\textit{Predicate} &\rightarrow \textit{Adverb}_{opt}\ \textit{VerbPhrase}
\end{aligned}
$$

Grammar, continued:

$$\textit{VerbPhrase} \rightarrow \begin{cases} \textit{TransitiveVerb NounPhrase} \\ \\ \textit{IntransitiveVerb} \\ \\ \textit{LinkingVerb Adjective} \end{cases}$$

$$\textit{PrepPhrase} \rightarrow \textit{Preposition NounPhrase}$$

Vocabulary:

Articles: `a the`

Adjectives: `big bright fast beautiful smart red smelly`

Nouns: `man woman dog unicorn ball field flea tree`

Adverbs: `quickly slowly happily dreamily`

Transitive verbs: `chased saw greeted bit loved`

Intransitive verbs: `ran slept sang`

Linking verbs: `was felt seemed`

Prepositions: `in on through with`

For next time:

Pg 74: 2.2.(11, 13, 15)
Pg 82: 2.4.(8-12, 14 & 15)

Read 3.(1-4)