

Where we are:

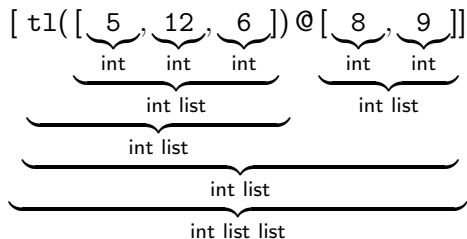
- ▶ Making types in SML (last week Wednesday)
- ▶ Functions in SML (last week Friday)
- ▶ Lists and functions on lists (Wednesday)
- ▶ Powersets; a language processor (**Today**)
- ▶ Propositional forms, logical equivalence [Start Chapter 3] (Friday)

Today:

- ▶ Powersets
 - ▶ Definition
 - ▶ Exploration
- ▶ A language processor
 - ▶ Case expressions and option types
 - ▶ The language processor itself
 - ▶ Introducing the semester project

Review

- ▶ List literals: [1, 4, 12, 3], []
- ▶ Analytic operations: hd, tl
- ▶ Synthetic operations: :: (cons), @ (cat)
- ▶ Lists vs tuples
- ▶ Type analysis problems :



- ▶ Lists as models for sets

Powersets

- ▶ Informal definition: The powerset of a set is the set of all subsets of that set.
- ▶ Formal definition: The powerset of a set X is

$$\mathcal{P}(X) = \{ Y \mid Y \subseteq X \}$$

- ▶ For “set of sets,” think “box of boxes.”
- ▶ Examples:

Why powersets seem to throw some people:

- ▶ The elements of a powerset are themselves sets.
- ▶ Suppose $X \subseteq \mathcal{U}$. Then
 - ▶ If $x \in X$, then $x \in \mathcal{U}$
 - ▶ $\mathcal{P}(X) \not\subseteq \mathcal{U}$, but rather $\mathcal{P}(X) \subseteq \mathcal{P}(\mathcal{U})$
 - ▶ If $A \in \mathcal{P}(X)$, then $A \in \mathcal{P}(\mathcal{U})$
- ▶ $\mathcal{P}(\emptyset) = \{\emptyset\} \neq \emptyset$. $|\emptyset| = 0$, but $|\{\emptyset\}| = 1$

Which are true?

$$\{3\} \in \mathcal{P}(\{1, 2, 3, 4, 5\})$$

$$3 \in \mathcal{P}(\{1, 2, 3, 4, 5\})$$

$$\{3\} \subseteq \mathcal{P}(\{1, 2, 3, 4, 5\})$$

$$3 \subseteq \mathcal{P}(\{1, 2, 3, 4, 5\})$$

$$a \in A \text{ iff } \{a\} \in \mathcal{P}(A)$$

$$a \in A \text{ iff } \{a\} \subseteq \mathcal{P}(A)$$

$$A \subseteq B \text{ iff } A \subseteq \mathcal{P}(B)$$

$$A \subseteq B \text{ iff } A \in \mathcal{P}(B)$$

Which are true?

$$A \subseteq B \text{ iff } A \subseteq \mathcal{P}(B)$$

$$\{A\} \subseteq \mathcal{P}(A)$$

$$A \in \mathcal{P}(A)$$

$$\{A\} \in \mathcal{P}(A)$$

$$\mathbb{Z} \in \mathcal{P}(\mathbb{R})$$

$$\emptyset = \mathcal{P}(\emptyset)$$

Note that

- ▶ $a \in A$ iff $\{a\} \in \mathcal{P}(A)$
- ▶ $A \subseteq B$ iff $A \in \mathcal{P}(B)$
- ▶ $A \subseteq B$ iff $\mathcal{P}(A) \subseteq \mathcal{P}(B)$
- ▶ $\mathcal{P}(\emptyset) = \{\emptyset\} \neq \emptyset$

Observe

$$\begin{aligned} \mathcal{P}(\{1, 2, 3\}) &= \{ \emptyset \\ &\quad \{1\}, \{2\}, \{3\} \\ &\quad \{1, 2\}, \{1, 3\}, \{2, 3\} \\ &\quad \{1, 2, 3\} \} \\ &= \{ \{1\}, \{1, 2\}, \{1, 3\}, \{1, 2, 3\} \\ &\quad \emptyset, \{2\}, \{3\}, \{2, 3\} \} \\ &= \mathcal{P}(\{2, 3\}) \cup \left[\begin{array}{l} \text{1 added to each set} \\ \text{of } \mathcal{P}(\{2, 3\}) \end{array} \right] = \mathcal{P}(\{2, 3\}) \cup \\ &\quad \{ \{1\} \cup X \mid X \in \mathcal{P}(\{2, 3\}) \} \end{aligned}$$

If $a \in A$, then $\mathcal{P}(A) = \mathcal{P}(A - \{a\}) \cup \{ \{a\} \cup X \mid X \in \mathcal{P}(A - \{a\}) \}$

What is $|\mathcal{P}(X)|$ in terms of $|X|$?

Grammar:

Sentence → *NounPhrase Predicate PrepPhrase_{opt}*

NounPhrase → *Article Adjective_{opt} Noun*

Predicate → *Adverb_{opt} VerbPhrase*

Grammar, continued:

VerbPhrase → { *TransitiveVerb NounPhrase*
IntransitiveVerb
LinkingVerb Adjective

PrepPhrase → *Preposition NounPhrase*

Vocabulary:

Articles: a the

Adjectives: big bright fast beautiful smart red smelly

Nouns: man woman dog unicorn ball field flea tree

Adverbs: quickly slowly happily dreamily

Transitive verbs: chased saw greeted bit loved

Intransitive verbs: ran slept sang

Linking verbs: was felt seemed

Prepositions: in on through with

For next time:

Pg 74: 2.2.(11, 13, 15)

Pg 82: 2.4.(8-12, 14 & 15)

Read 3.(1-4)