

Chapter 3:

- ▶ Propositions, booleans, logical equivalence. §3.(1–4) (**Today**)
- ▶ Conditional propositions, conditional expressions. §3.(5–7) (next week Monday)
- ▶ Arguments. §3.(8 & 9) (next week Wednesday)
- ▶ Predicates and quantification. §3.(10–13) (next week Friday)
- ▶ Quantified arguments. §3.14 (week-after Monday)

Today:

- ▶ Highlight main points of §3.(1&2): Propositions, forms, etc
- ▶ Demo SML features from §3.3: Boolean values
- ▶ Work through §3.4: Logical equivalences (Game 1)

Which phrase gives the best metaphor for the meaning of “set of sets”?

Champion of champions

Horror of horrors

Box of boxes

Friend of a friend

What is the cardinality of $\mathcal{P}(\emptyset)$?

If set X has cardinality n , then what is the cardinality of $\mathcal{P}(X)$?

A **proposition** is a sentence that is true or false, but not both.

It is snowing and it is not Thursday.

A **propositional form** is like a proposition but with content replaced by variables.

p and not q

$p \wedge \sim q$

$$\mathbb{Z} = \{\dots - 3, -2, -1, 0, 1, 2, 3 \dots\}$$

$+ - \times \div$

\times		0	1	2	3
0		0	0	0	0
1		0	1	2	3
2		0	2	4	6
3		0	3	6	9

$$\mathbb{B} = \{T, F\}$$

$\vee \wedge \sim$

\wedge		T	F
T		T	F
F		F	F

\wedge	T	F
T	T	F
F	F	F

\vee	T	F
T	T	T
F	T	F

p	$\sim p$
T	F
F	T

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

p	q	$p \wedge q$	$p \vee q$	$\sim p$
T	T	T	T	F
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

Evaluate (to T or F) this logical expression:

$$(T \wedge (\sim F \vee F)) \wedge (T \wedge T)$$

Evaluate (to T or F) this logical expression:

$$(T \vee F) \wedge \sim (F \wedge T)$$

Evaluate (to T or F) this logical expression:

$$(F \vee F \vee T) \wedge (\sim T \wedge F)$$

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim (p \wedge q)$	$\sim p \vee \sim q$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

Commutative laws:

$$p \wedge q \equiv q \wedge p$$

$$p \vee q \equiv q \vee p$$

Associative laws:

$$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$$

$$(p \vee q) \vee r \equiv p \vee (q \vee r)$$

Distributive laws:

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

Absorption laws:

$$p \wedge (p \vee q) \equiv p$$

$$p \vee (p \wedge q) \equiv p$$

Idempotent laws:

$$p \wedge p \equiv p$$

$$p \vee p \equiv p$$

Double negative law:

$$\sim \sim p \equiv p$$

DeMorgan's laws:

$$\sim (p \wedge q) \equiv \sim p \vee \sim q$$

$$\sim (p \vee q) \equiv \sim p \wedge \sim q$$

Negation laws:

$$p \vee \sim p \equiv T$$

$$p \wedge \sim p \equiv F$$

Universal bound laws:

$$p \vee T \equiv T$$

$$p \wedge F \equiv F$$

Identity laws:

$$p \wedge T \equiv p$$

$$p \vee F \equiv p$$

Tautology and
contradiction laws:

$$\sim T \equiv F$$

$$\sim F \equiv T$$

Remember from high school algebra that there are “simplify” problems and “solve” problems.

■ Simplify $3x(2 + 3x)^2 + 1$.

$$\begin{aligned} & 3x(2 + 3x)^2 + 1 \\ &= 3x(4 + 12x + 9x^2) + 1 \\ &= 12x + 36x^2 + 27x^3 + 1 \\ &= 27x^3 + 36x^2 + 12x + 1 \end{aligned}$$

■ Solve $12x = 57 - 7x$ for x .

$$\begin{aligned} 12x &= 57 - 7x \\ 19x &= 57 \\ x &= 3 \end{aligned}$$

Suppose we were to show that $\sim (\sim p \wedge q) \vee (p \vee \sim p) \equiv p \vee \sim q$.

Do this:

$$\begin{aligned} & \sim (\sim p \wedge q) \vee (p \wedge \sim p) \\ \equiv & \sim (\sim p \wedge q) \vee F && \text{by negation law} \\ \equiv & \sim (\sim p \wedge q) && \text{by identity law} \\ \equiv & p \vee \sim q && \text{by De Morgan's} \end{aligned}$$

Don't do this:

$$\begin{aligned} \sim (\sim p \wedge q) \vee (p \wedge \sim p) & \equiv p \vee \sim q \\ \sim (\sim p \wedge q) \vee F & \equiv p \vee \sim q && \text{by negation law} \\ \sim (\sim p \wedge q) & \equiv p \vee \sim q && \text{by identity law} \\ p \vee \sim q & \equiv p \vee \sim q && \text{by De Morgan's} \end{aligned}$$

Semester roadmap:

Ch 1 & 2: Raw materials

Ch 3: Formal logic

—Test 1, Feb 10 —

Ch 4: Proofs

Ch 5: Relations

— Test 2, Mar 20 —

Ch 6: Self reference

Ch 7: Functions

— Test 3, Apr 19 —

Chapter 3 roadmap:

Today: Logical equivalences (Game 1)

Next week Monday: Conditionals (SML)

Next week Wednesday: Arguments (Game 2)

Next week Friday: Predicates and quantification (SML)

Week-after Monday: Quantified arguments (Game 3)

Week-after Wednesday: Review for test

For next time:

Pg 102: 3.3.(5 & 6)

Pg 105: 3.4.(2, 4, 8-12)

(See Schoology for a note about 3.4.(2 & 4))

Read 3.(5-7)

Take quiz