Semester roadmap:

Ch 1 & 2: Raw materials
Ch 3: Formal logic
—Test 1, Feb 10 —
Ch 4: Proofs
Ch 5: Relations
— Test 2, Mar 20 —
Ch 6: Self reference
Ch 7: Functions
— Test 3, Apr 19—

Chapter 6 roadmap:

- Recursive definitions, recursive types (**Today**)
- Recursive proofs I: Structural induction (Friday)
- Recursive proofs II: Mathematical induction (next week Monday)
- Recursive proofs III: Loop invariants (next week Wednesday and Friday)

Project prototype due Fri, Mar 31

Axiom 7

There exists a whole number 0.

Axiom 8

Every whole number n has a successor, $\texttt{succ}\ n$.

Axiom 9

No whole number has 0 as its successor.

Axiom 10

If $a, b \in \mathbb{W}$, then $a = b$ iff $\texttt{succ}\ a = \texttt{succ}\ b$.

A *whole number* is either zero or one more than another whole number.

Compare to:
A *list* is either empty or an element together with its following list.

5 is a whole number because

5 is a whole number because it is the successor of 4, which is a whole number because

5 is a whole number because it is the successor of

4, which is a whole number because it is the successor of

3, which is a whole number because

5 is a whole number because it is the successor of

4, which is a whole number because it is the successor of

3, which is a whole number because it is the successor of

2, which is a whole number because

5 is a whole number because it is the successor of

4, which is a whole number because it is the successor of

3, which is a whole number because it is the successor of

2, which is a whole number because it is the successor of

1, which is a whole number because

5 is a whole number because it is the successor of

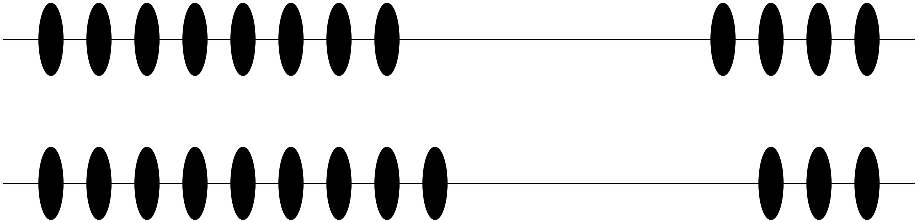
4, which is a whole number because it is the successor of

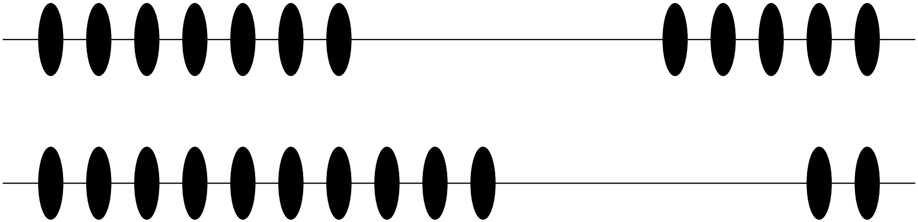
3, which is a whole number because it is the successor of

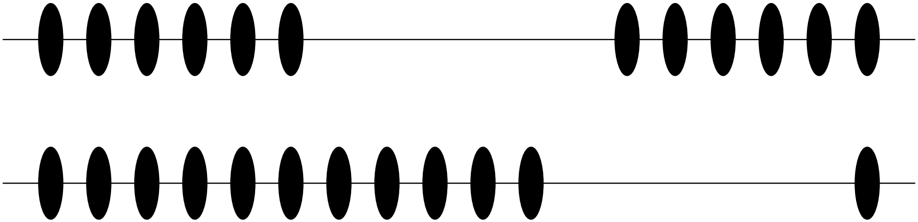
2, which is a whole number because it is the successor of

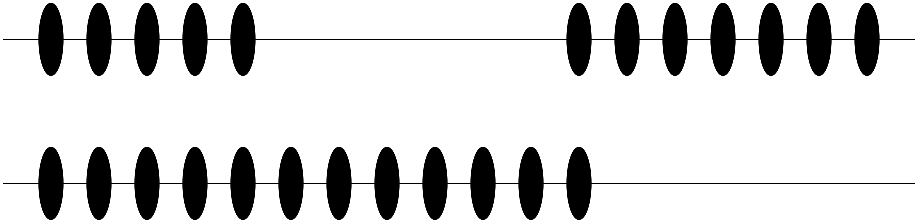
1, which is a whole number because it is the successor of

0, which is a whole number by Axiom 7.









Lemmas for addition:

- $0 + b = b$
- $a + 0 = a$
- $a + b = (a + 1) + (b - 1)$

Lemmas for subtraction:

- $a - 0 = a$
- $a - b = (a - 1) - (b - 1)$

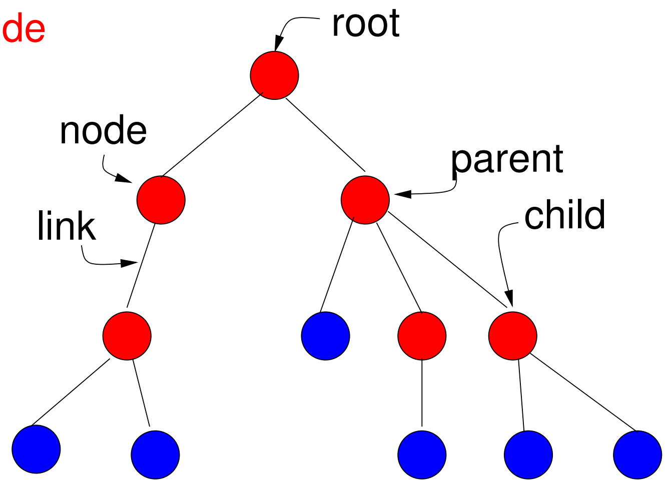
Lemmas for multiplication:

- $a \cdot 0 = 0$
- $0 \cdot b = 0$
- $a \cdot 1 = a$
- $a \cdot b = a + (a \cdot (b - 1))$

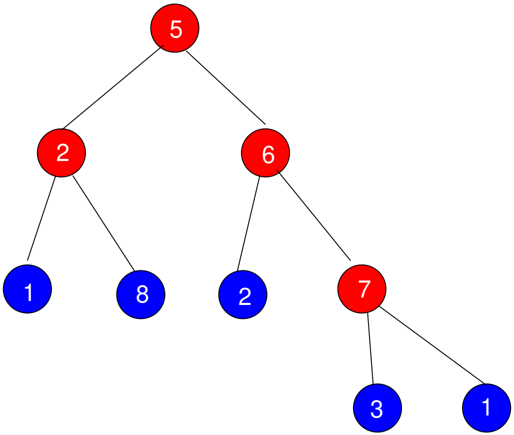
Tree

internal node

leaf



Full Binary Tree

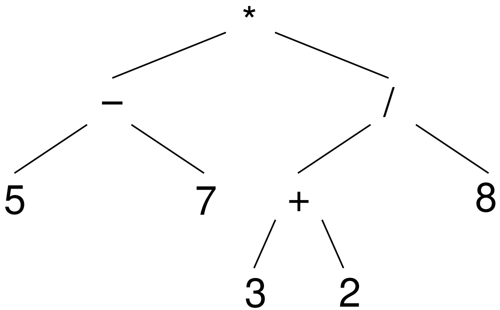


Expression trees:

```
datatype operation = Plus | Minus | Mul | Div;
datatype expression =  Internal of operation * expression * expression
      | Leaf of int;
```

$((5-7)*((3+2)/8))$

```
val exprExample = Internal(Mul, Internal(Minus,Leaf(5), Leaf(7)),
                                Internal(Div,
                                         Internal(Plus, Leaf(3),
                                                        Leaf(2)),
                                         Leaf(8)));
```



For next time:

Pg 260: 6.2.(6-8, 14-17)

Read 6.4