

Chapter 3, Case Studies:

- ▶ Linear-time sorting algorithms (last week Monday and Wednesday)
- ▶ Disjoint sets and array forests (last week Friday)
- ▶ Priority queues (**Today**)
- ▶ N -sets and bit vectors (Wednesday)
- ▶ (Start graphs Friday)

Today:

- ▶ Recent HW problems
- ▶ Worklist algorithms
- ▶ Priority queue ADT (problem statement)
- ▶ Inefficient solutions
- ▶ Abstractions for the heap data structure
- ▶ Heap implementation details, part 1
- ▶ Excursion: heap sort
- ▶ Heap implementation details, part 2
- ▶ Analysis and optimization

```
public class ArrayList<E>
    implements List<E> {
    private E[] internal;
    private int size;
    ...
}
```

```
public class ArrayStack<E>
    implements Stack<E> {
    private E[] internal;
    private int size;
    ...
}
```

Informal: The positions of `internal` in range $[0, \text{size})$ contain the elements in the conceptual list in order.

Formal:

- (a) $\text{size} \leq \text{internal.length}$
- (b) `size` equals the number of calls to `insert()` and `add()` minus the number of (non-empty) calls to `remove()`.

Informal: `size` is the number of items in the conceptual stack, which appear in order of arrival in range $[0, \text{size})$

Formal:

- (a) $\text{size} \leq \text{internal.length}$
- (b) `size` equals the number of calls to `push()` minus the number of (non-empty) calls to `pop()`.
- (c) For all $i \in [0, \text{size} - 1)$, the item in `internal[i]` was pushed before the item in `internal[i + 1]`.

```

private class PlainFind implements FindStrategy {
    public int find(int p) {
        while (p != parents[p]) p = parents[p];
        return p;
    }
}

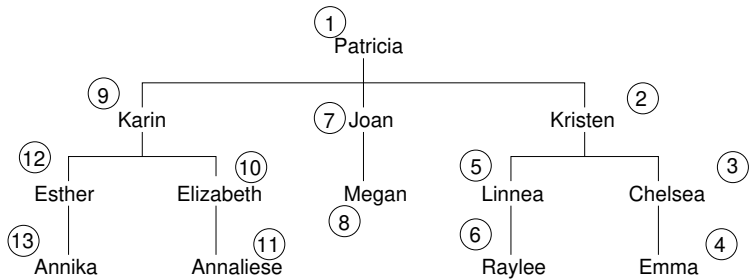
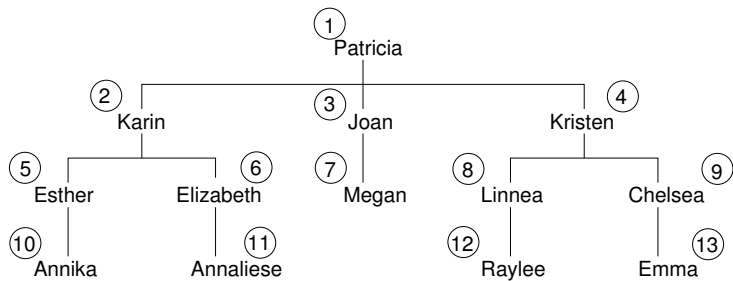
```

Let p_{orig} be the original value of p . Let i be the number of iterations completed.

(a) $0 \leq p < N$.

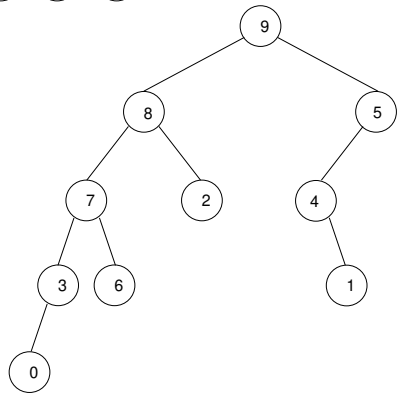
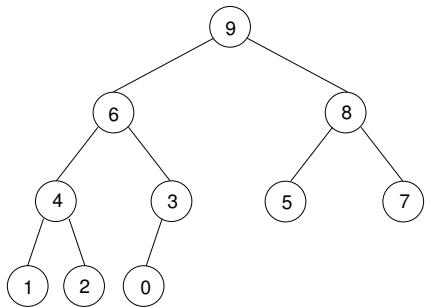
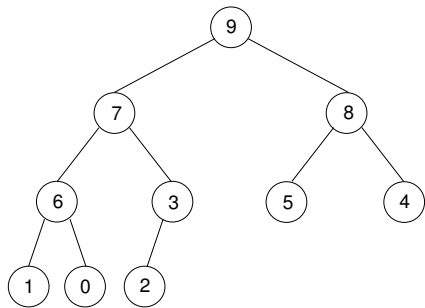
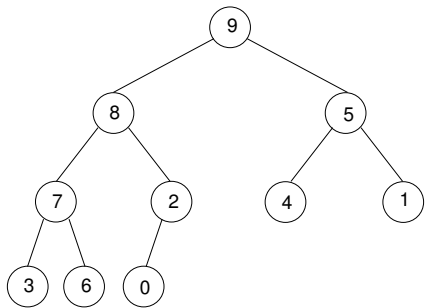
(b) $\text{leader}(p) = \text{leader}(p_{\text{orig}})$

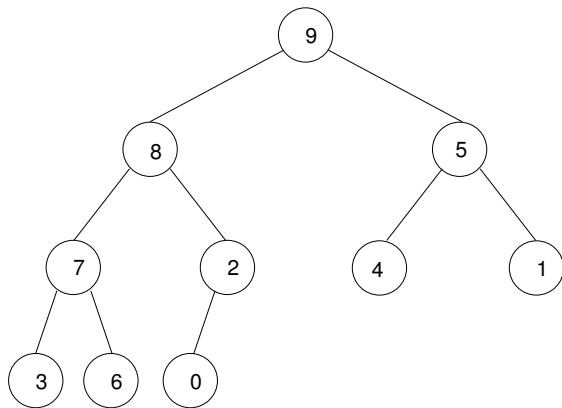
(c) $p = \overbrace{p}^{i \text{ times}}[p_{\text{orig}}]$



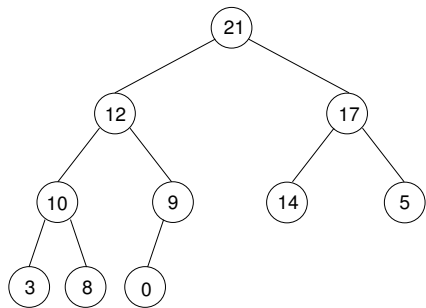
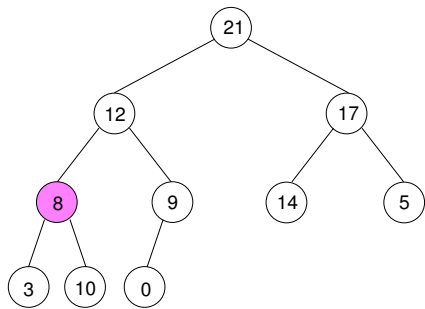
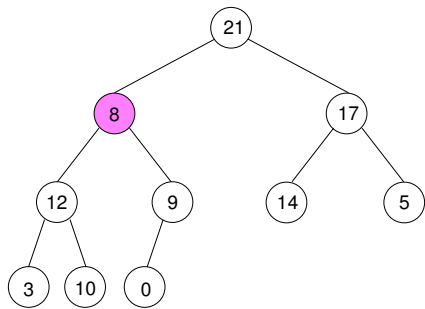
	ListPriorityQueue	SortedListPriorityQueue
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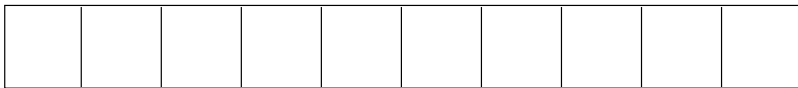
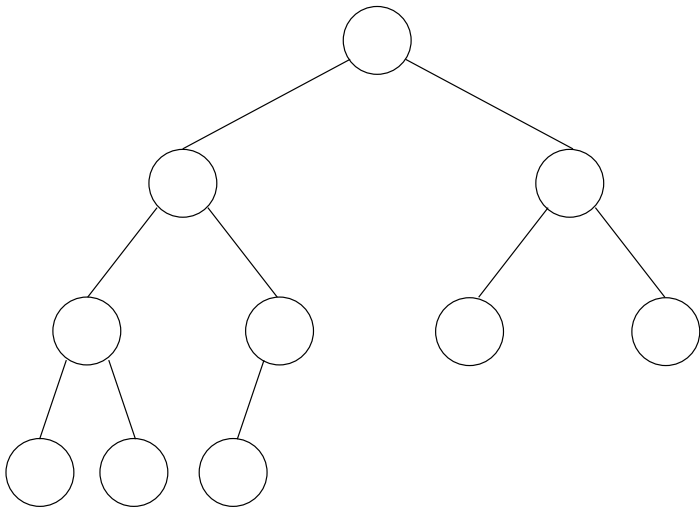
Initialize empty	$\Theta(1)$	$\Theta(1)$
Initialize populated	$\Theta(n)$	$\Theta(n^2)$
insert	$\Theta(1)$	$\Theta(n)$
max	$\Theta(n)$	$\Theta(1)$
extractMax	$\Theta(n)$	$\Theta(1)$
contains	$\Theta(n)$	$\Theta(n)$
increaseKey	$\Theta(1)$	$\Theta(n)$
decreaseKey	$\Theta(1)$	$\Theta(n)$



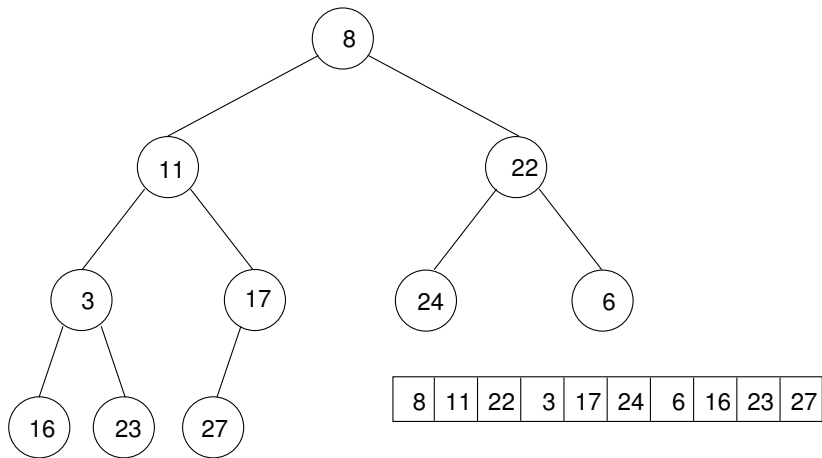


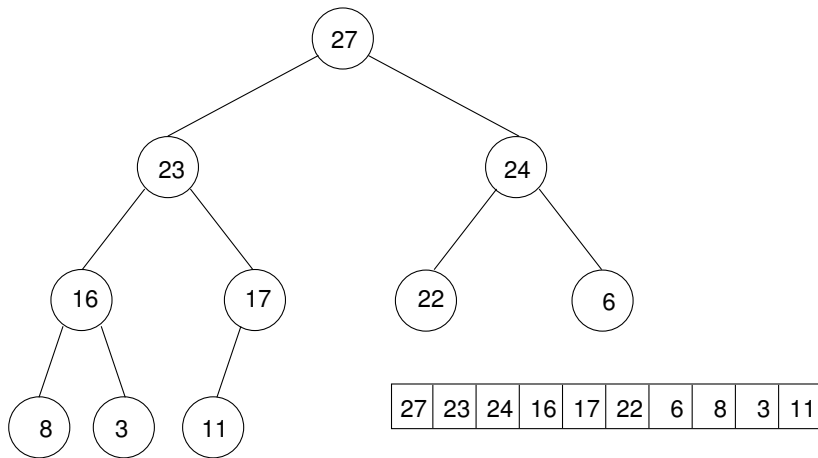
9	8	5	7	2	4	1	3	6	0
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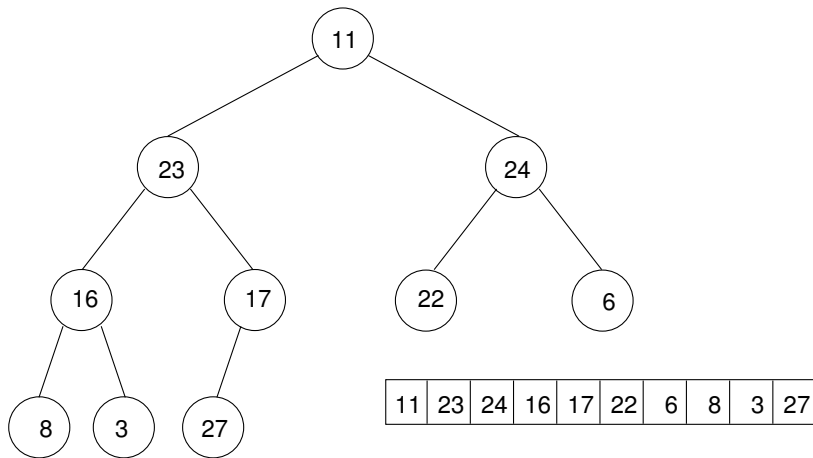


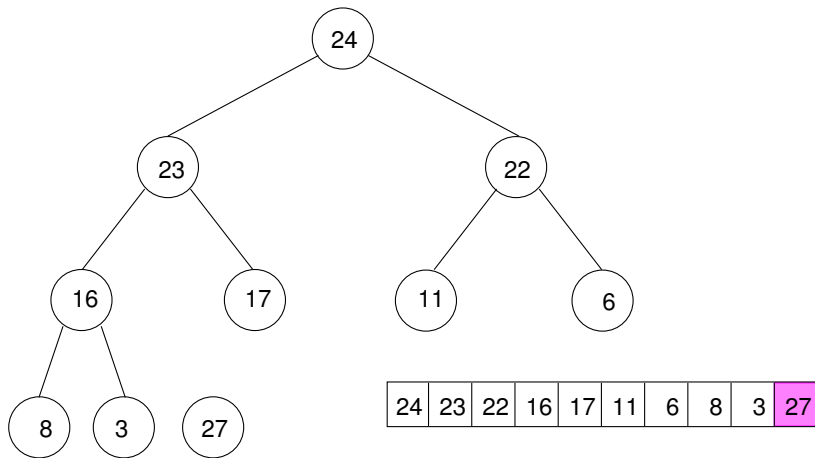


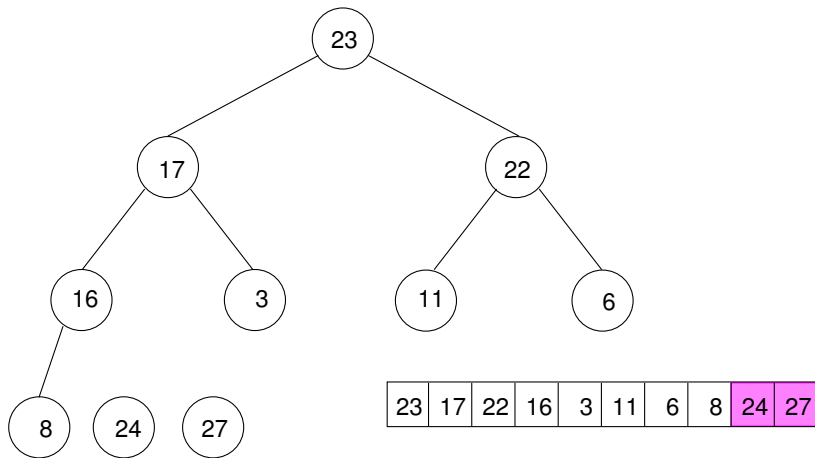
$$\begin{aligned}\sum_{i=0}^{h-1} 2^i (h-1-i) &= (h-1) \sum_{i=0}^{h-1} 2^i - \sum_{i=0}^{h-1} i 2^i \\ &= (h-1)(2^h - 1) - 2 - (h-2)2^h \\ &= h2^h - 2^h - h + 1 - 2 - h2^h + 2 \cdot 2^h \\ &= 2^h - h - 1 \\ &= 2^{\lg(n+1)} - \lg(n+1) - 1 \\ &= n + 1 - \lg(n+1) - 1 \\ &= n - \lg(n+1)\end{aligned}$$

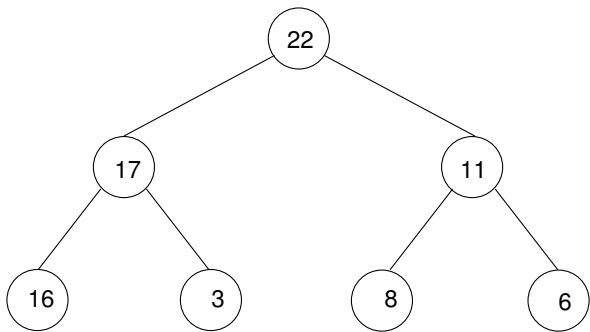




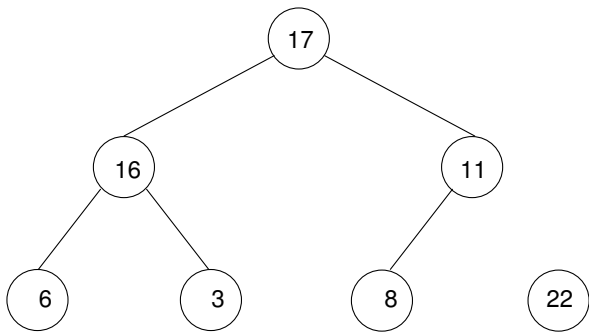




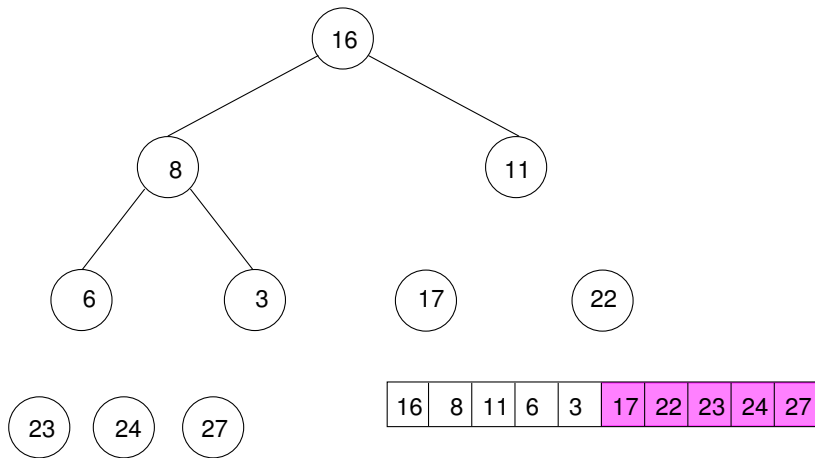


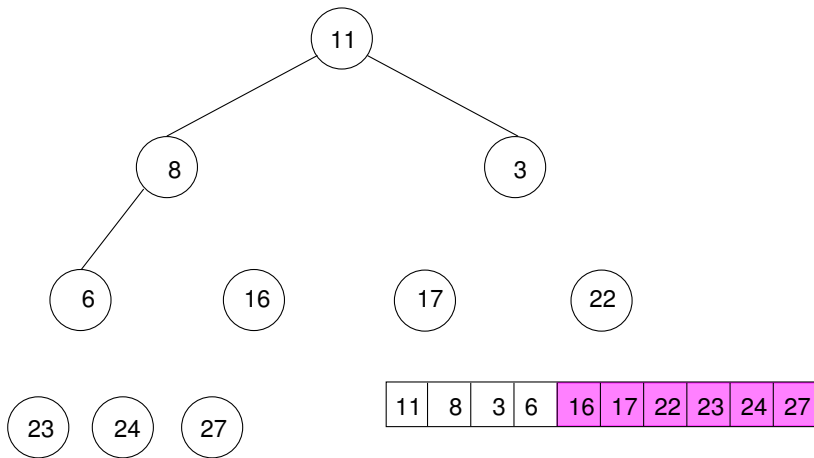


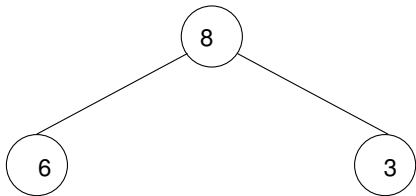
22	17	11	16	3	8	6	23	24	27
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17	16	11	6	3	8	22	23	24	27
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11

16

17

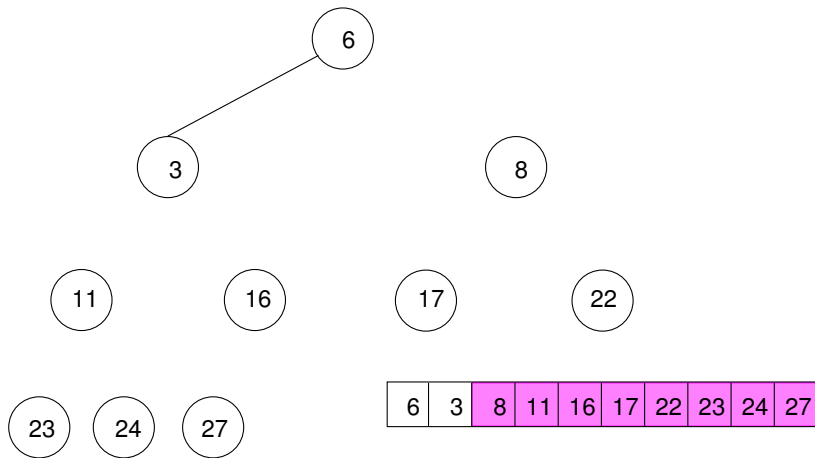
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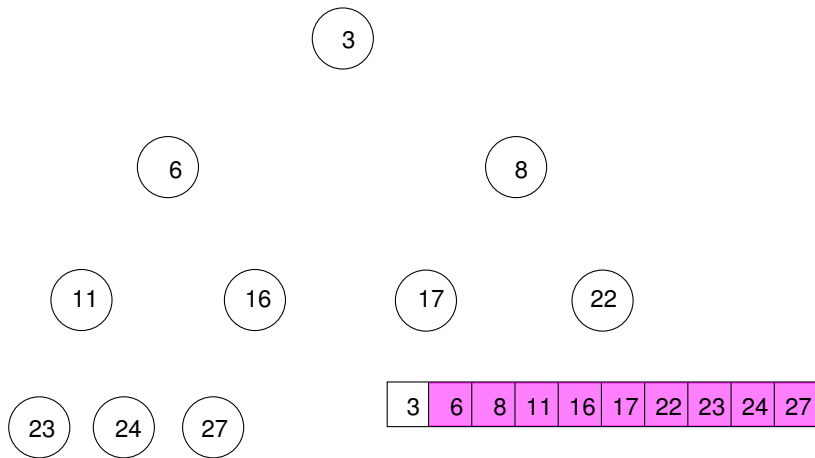
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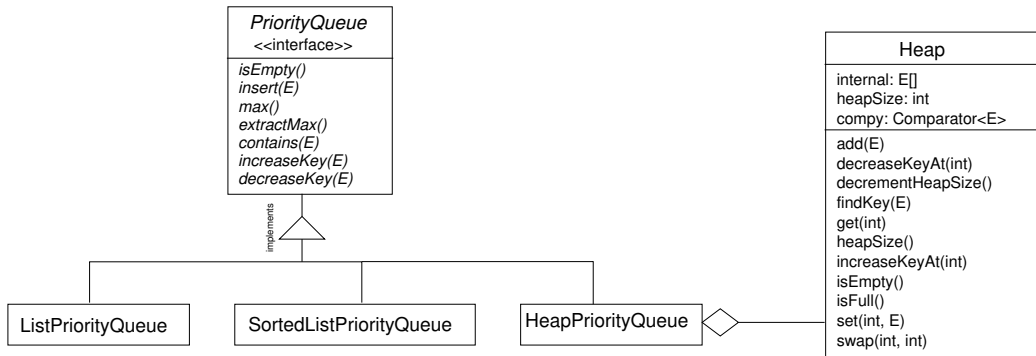
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27

8	6	3	11	16	17	22	23	24	27
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	ListPriorityQueue	SortedPriorityQueue	HeapPriorityQueue
Initialize empty	$\Theta(1)$	$\Theta(1)$	$\Theta(1)$
Initialize populated	$\Theta(n)$	$\Theta(n^2)$	$\Theta(n)$
insert	$\Theta(1)$	$\Theta(n)$	$\Theta(\lg n)$
max	$\Theta(n)$	$\Theta(1)$	$\Theta(1)$
extractMax	$\Theta(n)$	$\Theta(1)$	$\Theta(\lg n)$
contains	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$
increaseKey	$\Theta(1)$	$\Theta(n)$	$\Theta(n)$
decreaseKey	$\Theta(1)$	$\Theta(n)$	$\Theta(n)$

Coming up: (all end-of-day)

Do **linear sorting** project (*suggested by today*)

Do **heaps and priority queue** project (*suggested by Mon, Feb 13*)

Due **Tuesday:**

Read Section 3.3 (heaps and priority queues)

Take heap/pq quiz

Due **Thursday:**

Read Section 3.4

Do Exercises 3.(26 & 27).

Take N-sets quiz