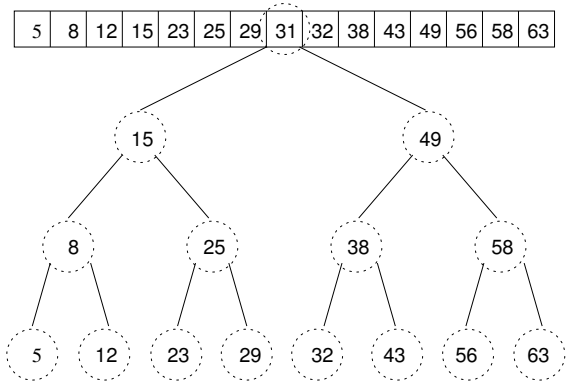
Chapter 5, Binary search trees:

- Binary search trees; the balanced BST problem (spring-break eve)
- AVL trees (last week Monday and Wednesday)
- Traditional red-black trees (last week Friday, finished Monday)
- Left-leaning red-black trees (Monday, finish today)
- "Wrap-up" BSTs, B-trees (**Today**)
- Begin dynamic programming (Friday)
- Test 2 Wednesday, Apr 5

Today:

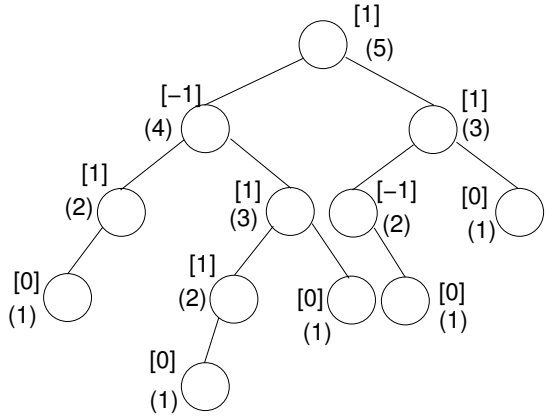
- Look ahead to Test 2
- Ex 5.(1 & 14)
- Finish left-learning RB cases
- Balanced tree comparisons
- Quick mention of B-trees

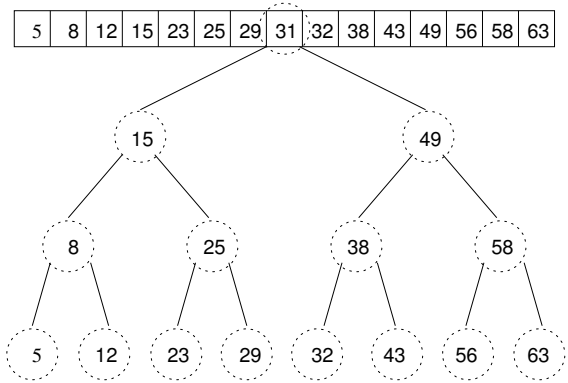
5.1. Write a method `bst2Array()` that takes a simplified BST represetned by the root node and returns a sorted array containing the keys.

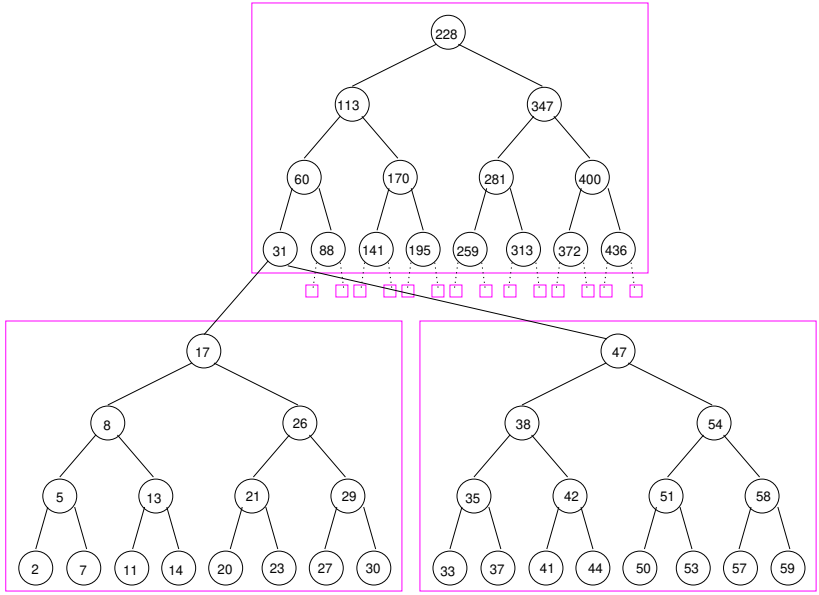


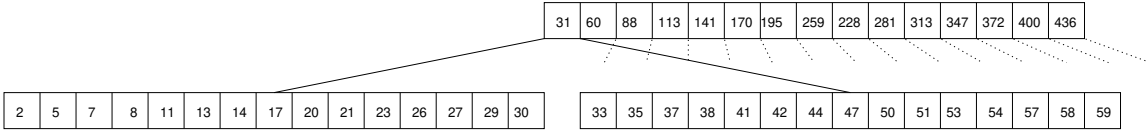
5.14. Write a method `AVLToRB.avl2rb()`, which takes an AVL tree represented by the `AVLNode` root and returns a `RBNode` that is the root to a red-black tree equivalent in keys and structure to the given AVL tree.

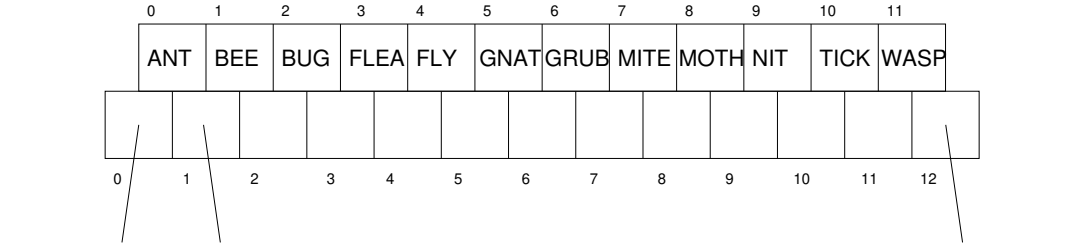
(Try it on this sample AVL tree.)




5 8 12 15 23 25 29 31 32 38 43 49 56 58 63
15
49
8
25
38
58
5
12
23
29
32
43
56
63

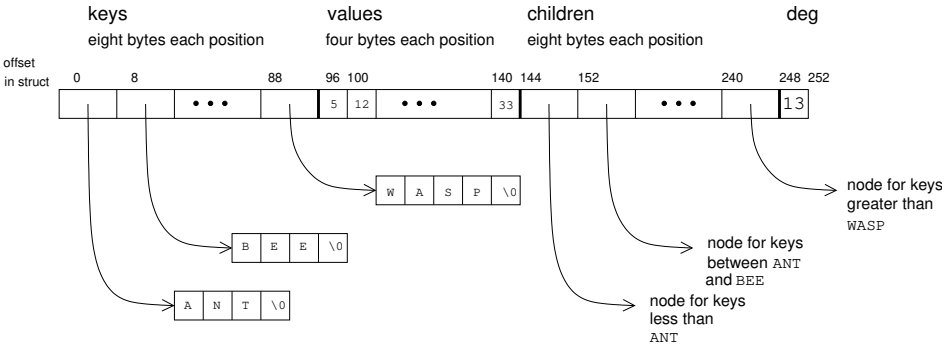

228
113
347
60
170
281
400
31
88
141
195
259
313
372
436
17
47
8
26
38
54
5
13
21
29
35
42
51
58
2
7
11
14
20
23
27
30
33
37
41
44
50
53
57
59

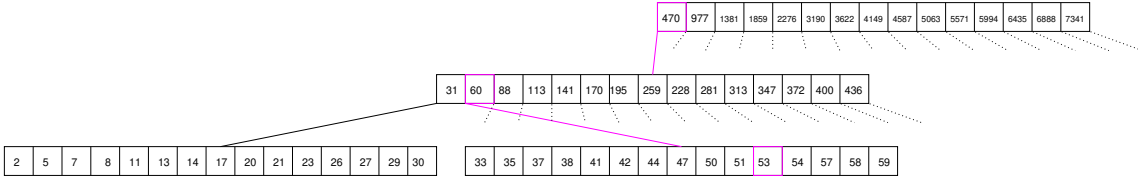

31 60 88 113 141 170 195 259 228 281 313 347 372 400 436
2 5 7 8 11 13 14 17 20 21 23 26 27 29 30
33 35 37 38 41 42 44 47 50 51 53 54 57 58 59

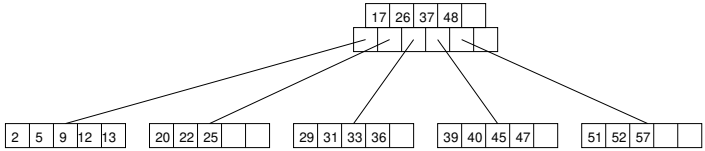

0
1
2
3
4
5
6
7
8
9
10
11
ANT
BEE
BUG
FLEA
FLY
GNAT
GRUB
MITE
MOTH
NIT
TICK
WASP
0
1
2
3
4
5
6
7
8
9
10
11
12
Subtree with keys less than ANT
Subtree with keys between ANT and BEE
Subtree with keys greater than WASP

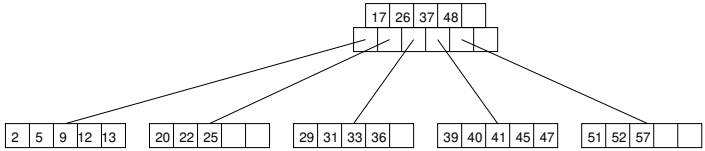
Formally, a B-tree with maximum degree M over some ordered key type is either

- empty, or
- a node with with $d-1$ keys and d children, designated as lists `keys` and `children` such that
  - $\lceil M/2 \rceil \leq d \leq M$,
  - `children`$[0]$ is a B-tree such that all of the keys in that tree are less than `keys`$[0]$,
  - for all $i \in [1, d-1)$, `children`$[i]$ is a B-tree such that all of the keys in that tree are greater than `keys`$[i-1]$ and less than `keys`$[i]$,
  - and `children`$[d-1]$ is a B-tree such that all of the keys in that tree are greater than `keys`$[d-2]$.


keys
eight bytes each position
values
four bytes each position
children
eight bytes each position
deg
offset
in struct
0
8
88
96
100
140
144
152
240
248
252
5
12
33
13
W A S P \0
B E E \0
A N T \0
node for keys
greater than
WASP
node for keys
between ANT
and BEE
node for keys
less than
ANT


470 977 1381 1859 2276 3190 3622 4149 4587 5063 5571 5994 6435 6888 7341
31 60 88 113 141 170 195 259 228 281 313 347 372 400 436
2 5 7 8 11 13 14 17 20 21 23 26 27 29 30
33 35 37 38 41 42 44 47 50 51 53 54 57 58 59


17 26 37 48
2 5 9 12 13
20 22 25
29 31 33 36
39 40 45 47
51 52 57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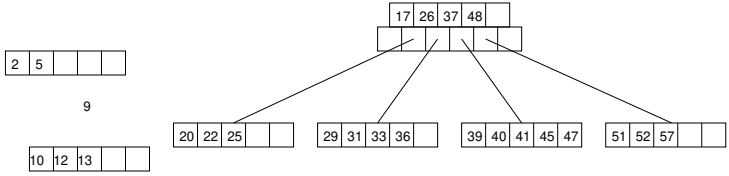
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29 31 33 36
39 40 41 45 47
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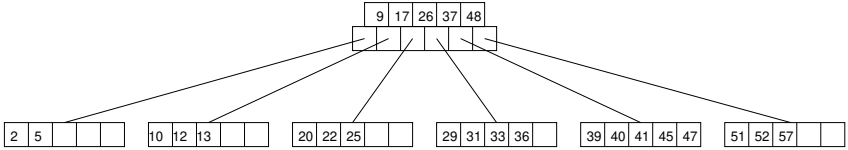
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10 12 13
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29 31 33 36
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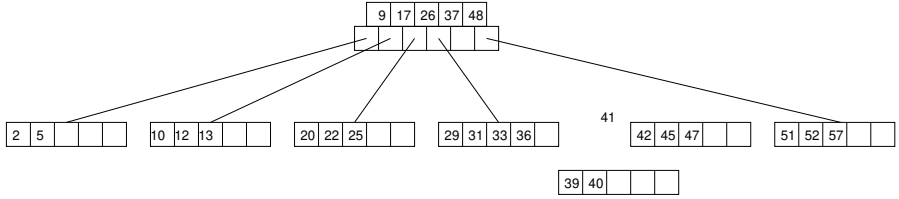
51 52 57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42 45 47
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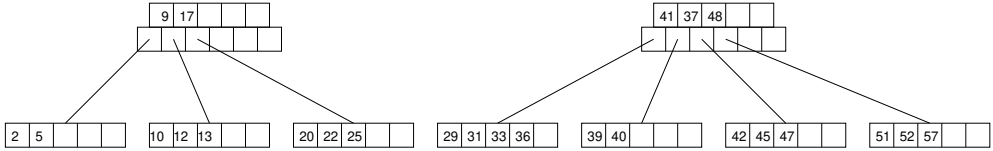
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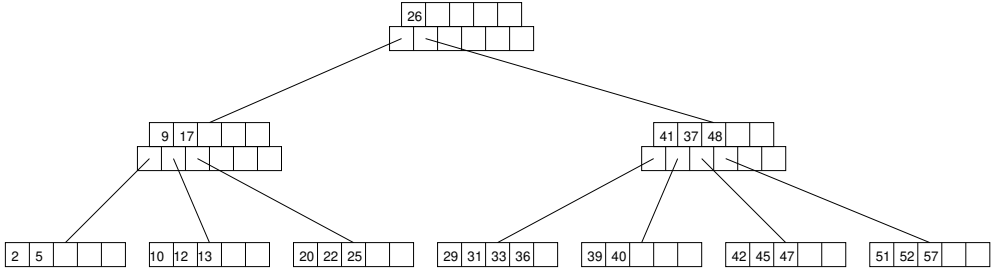
26
9 17
41 37 48
2 5
10 12 13
20 22 25
29 31 33 36
39 40
42 45 47
51 52 57

$$\underbrace{(M-1)}_{\substack{\text{keys per}\\ \text{node}}} \underbrace{\sum_{i=0}^{h-1} M^i}_{\substack{\text{sum of nodes}\\ \text{at each}\\ \text{level}}} = (M-1)\frac{M^h-1}{M-1} = M^h - 1$$

$$\begin{aligned} n &= M^h - 1 \\ M^h &= n+1 \\ h &= \log_M(n+1) \end{aligned}$$

$$
\begin{aligned}
n &= M^h - 1 \\
M^h &= n + 1 \\
h &= \log_M(n+1)
\end{aligned}
$$

$$
h = \log_{\frac{M}{2}}(n+1) = \frac{\log_M(n+1)}{1 - \log_M 2}
$$

Cost of a search:

$$\begin{aligned}
\lg M \cdot h &= \lg M \cdot \frac{\log_M(n+1)}{1-\log_M 2} \\
&= \lg M \frac{\frac{\lg(n+1)}{\lg M}}{1-\frac{\lg 2}{\lg M}} \\
&= \frac{\lg(n+1)}{1-\frac{1}{\lg M}} \\
&= \frac{\lg M}{\lg M-1} \lg(n+1)
\end{aligned}$$

Compare: $1.44 \lg n$ for AVL trees, $2 \lg n$ for RB trees.

Let c_0 be the cost of searching at a node (proportional to $\lg M$) and c_1 be the cost of reading a node from memory. The the cost of an entire search is

$$(c_0 + c_1)\frac{\log_M(n+1)}{1 - \log_M 2}$$

Now, consolidate the constants by letting $d = \frac{c_0+c_1}{1-\log_M 2}$, and we have

$$d \log_M(n+1)$$

Coming up:

Do **Traditional RB** *project (suggested by Mon, Mar 27)*
(Recommended: Do **Left-leaning RB** *project for your own practice)*

Due **Wed, Mar 22 (today** *(end of day) (but hopefully you've spread it out)*
Read Sections 5.(4-6) [some parts carefully, some parts skim, some parts optional—see Schoology]
Do Exercise 5.14 (which we've already seen a solution for)
Take quiz

Due **Mon, Mar 27** *(class time)*
Read Section 6.(1&2)
Do Exercises 6.(5–7)
Take quiz

Due **Tues, Mar 28** *(end of day)*
Read Section 6.3
Do Exercises 6.(16, 19, 23, 33)
Take quiz