Chapter 4 roadmap:

- Subset proofs (last week Wednesday)
- Set equality and emptiness proofs (last week Friday)
- Conditional and biconditional proofs (**Today**)
- Proofs about powersets (Friday)
- From theorems to algorithms (next week Monday)
- (Start Chapter 5 next week)

Today:

- Proofs of conditional propositions
- Proofs about numbers
- Proofs of biconditional propositions

General forms:

1. Facts (p)
   Set forms
   1. Subset $X \subseteq Y$
   2. Set equality $X = Y$
   3. Set emptiness $X = \emptyset$
2. Conditionals ($p \rightarrow q$)
3. Biconditionals ($p \leftrightarrow q$)

Hypothetical conditional from **Game 3**:

To prove $p \to q$

Suppose p

$\dots$

q

$p \to q$

An integer x is *even* if $\exists\ k \in \mathbb{Z} \mid x = 2k$.

An integer x is *odd* if $\exists\ k \in \mathbb{Z} \mid x = 2k + 1$.

"Axiom 3." If $x, y \in \mathbb{Z}$, then $x + y \in \mathbb{Z}$. (*Closure of addition*)

"Axiom 4." If $x, y \in \mathbb{Z}$, then $x \cdot y \in \mathbb{Z}$. (*Closure of multiplication*)

"Axiom 5." If $x \in \mathbb{Z}$, then x is even iff x is not odd.

$\forall\ x, y \in \mathbb{Z}$, $x \mid y$ (read, "x divides y") if $\exists\ k \in \mathbb{Z} \mid x \cdot k = y$.

Note that $y/x = k$ or $\frac{y}{x} = k$ or $x \overset{\displaystyle k}{\overline{\big)\, y}}$.

For next time:

Pg 162: 4.5.(1, 4, 5)
Pg 164: 4.6.(2 & 5)
Pg 165: 4.7.(1 & 6)

Review 2.4, especially Ex 2.4.15
Skim 4.9

Take quiz