Chapter 1 & 2 outline:

- Introduction, sets and elements (last week Monday)
- Set operations; visual verification of set propositions (last week Wednesday)
- Introduction to SML; cardinality and Cartesian products (last week Friday)
- Making types in SML (this week Wednesday)
- Functions in SML (last week Friday)
- Functions on lists (**Today**)
- Powersets; a language processor (Friday)
- (Begin chapter 3, Propositions, next week Monday)

Today:

- Review of lists
- Type analysis of lists
- Functions on lists
- (Time permitting) Begin powersets

```
[tl([5, 12, 6])@[8, 9]]
```

```
hd([12, 5, 6])::[2, 7]
```

```
[[(2.3, 5), (8.1, 6)],[]]
```

```
([1, 12, 81], ["a", "bc"])
```

## Powersets

- Informal definition: The powerset of a set is the set of all subsets of that set.
- Formal definition: The powerset of a set $X$ is

$$\mathscr{P}(X) = \{ \ Y \mid Y \subseteq X \}$$

- For "set of sets," think "box of boxes."
- Examples:

Why powersets seem to throw some people:

- The elements of a powerset are themselves sets.
- Suppose $X \subseteq \mathcal{U}$. Then
  - If $x \in X$, then $x \in \mathcal{U}$
  - $\mathscr{P}(X) \nsubseteq \mathcal{U}$, but rather $\mathscr{P}(X) \subseteq \mathscr{P}(\mathcal{U})$
  - If $A \in \mathscr{P}(X)$, then $A \in \mathscr{P}(\mathcal{U})$
- $\mathscr{P}(\emptyset) = \{\emptyset\} \neq \emptyset$. $|\emptyset| = 0$, but $|\{\emptyset\}| = 1$

**For next time:**

*If you had trouble on the programming problems from last time, ask for help and try again.*
*Pg 70: 2.1.(2-4, 9, 10) [on paper]*
*Pg 74: 2.2.(2, 3, 8, 9, 11) [through turn-in page]*

*See notes on Ex 2.2.8 and 2.2.9 on the Canvas description of the assignment for clarifications and hints. See also the code from class for "starter code."*
*You do* **not** *need to include your SML code with your on-paper problems that you turn in.*

*Read 2.(4 & 5)*
*Take quiz*
*(There will be a follow-up quiz after class Friday)*