Chapter 3:

- Propositions, booleans, logical equivalence. §3.(1–4) (**Today**)
- Conditional propositions, conditional expressions. §3.(5–7) (Wednesday)
- Arguments. §3.(8 & 9) (Friday)
- Predicates and quantification. §3.(10–13) (next week Monday)
- Quantified arguments. §3.14 (next week Wednesday)

Today:

- Highlight main points of §3.(1&2): Propositions, forms, etc
- Demo SML features from §3.3: Boolean values
- Work through §3.4: Logical equivalences (Game 1)

Which phrase gives the best metaphor for the meaning of “set of sets”?

Champion of champions	Horror of horrors
Box of boxes	Friend of a friend

What is the cardinality of $\mathscr{P}(\emptyset)$?

If set X has cardinality n, then what is the cardinality of $\mathscr{P}(X)$?

A **proposition** is a sentence that is true or false, but not both.

It is snowing and it is not Thursday.

A **propositional form** is like a proposition but with content replaced by variables.

p and not q

$p \wedge \sim q$

$$\mathbb{Z} = \{\ldots -3, -2, -1, 0, 1, 2, 3 \ldots\}$$

$$+ - \times \div$$

$\times$	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	4	6
3	0	3	6	9

$$\mathbb{B} = \{T, F\}$$

$$\vee \wedge \sim$$

$\wedge$	T	F
T	T	F
F	F	F

$\wedge$	T	F
T	T	F
F	F	F

$\vee$	T	F
T	T	T
F	T	F

p	$\sim p$
T	F
F	T

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

p	q	$p \wedge q$	$p \vee q$	$\sim p$
T	T	T	T	F
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

Evaluate (to T or F) this logical expression:

$(T \wedge (\sim F \vee F)) \wedge (T \wedge T)$

Evaluate (to T or F) this logical expression:

$(T \vee F) \wedge \sim (F \wedge T)$

Evaluate (to T or F) this logical expression:

$(F \vee F \vee T) \wedge (\sim T \wedge F)$

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim (p \wedge q)$	$\sim p \vee \sim q$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

Commutative laws:	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
Associative laws:	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
Distributive laws:	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
Absorption laws:	$p \wedge (p \vee q) \equiv p$	$p \vee (p \wedge q) \equiv p$
Idempotent laws:	$p \wedge p \equiv p$	$p \vee p \equiv p$
Double negative law:	$\sim\sim p \equiv p$	
DeMorgan's laws:	$\sim (p \wedge q) \equiv \sim p \vee \sim q$	$\sim (p \vee q) \equiv \sim p \wedge \sim q$
Negation laws:	$p \vee \sim p \equiv T$	$p \wedge \sim p \equiv F$
Universal bound laws:	$p \vee T \equiv T$	$p \wedge F \equiv F$
Identity laws:	$p \wedge T \equiv p$	$p \vee F \equiv p$
Tautology and contradiction laws:	$\sim T \equiv F$	$\sim F \equiv T$

Remember from high school algebra that there are "simplify" problems and "solve" problems.

- Simplify $3x(2+3x)^2+1$.

$$\begin{aligned} & 3x(2+3x)^2+1 \\ = \quad & 3x(4+12x+9x^2)+1 \\ = \quad & 12x+36x^2+27x^3+1 \\ = \quad & 27x^3+36x^2+12x+1 \end{aligned}$$

- Solve $12x = 57 - 7x$ for x.

$$\begin{aligned} 12x &= 57-7x \\ 19x &= 57 \\ x &= 3 \end{aligned}$$

Suppose we were to show that $\sim(\sim p \wedge q) \vee (p \vee \sim p) \equiv p \vee \sim q$.

Do this:

$$\begin{aligned}
&\sim(\sim p \wedge q) \vee (p \wedge \sim p) && \\
\equiv\ & \sim(\sim p \wedge q) \vee F && \text{by negation law} \\
\equiv\ & \sim(\sim p \wedge q) && \text{by identity law} \\
\equiv\ & p \vee \sim q && \text{by De Morgan's}
\end{aligned}$$

Don't do this:

$$\begin{aligned}
\sim(\sim p \wedge q) \vee (p \wedge \sim p) &\equiv p \vee \sim q && \\
\sim(\sim p \wedge q) \vee F &\equiv p \vee \sim q && \text{by negation law} \\
\sim(\sim p \wedge q) &\equiv p \vee \sim q && \text{by identity law} \\
p \vee \sim q &\equiv p \vee \sim q && \text{by De Morgan's}
\end{aligned}$$

Semester roadmap:

Ch 1 & 2: Raw materials
Ch 3: Formal logic
—Test 1, Feb 12 —
Ch 4: Proofs
Ch 5: Relations
— Test 2, Mar 20 —
Ch 6: Self reference
Ch 7: Functions
— Test 3, Apr 19 —

Chapter 3 roadmap:

Today: Logical equivalences (Game 1)

Wednesday: Conditionals (SML)

Friday: Arguments (Game 2)

Next week Monday: Predicates and quantification (SML)

Next week Wednesday: Quantified arguments (Game 3)

Next week Friday: Review for test

For next time:

Pg 102: 3.3.(5 & 6)
Pg 105: 3.4.(2, 4, 8-12)
(See Canvas for a note about 3.4.(2 & 4))

Read 3.(5-7)
Take quiz