Chapter 3:

- Propositions, booleans, logical equivalence. §3.(1–4) (Today)
- Conditional propositions, conditional expressions. §3.(5–7) (Wednesday)

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- Arguments. §3.(8 & 9) (Friday)
- Predicates and quantification. §3.(10–13) (next week Monday)
- Quantified arguments. §3.14 (next week Wednesday)

Today:

- ▶ Highlight main points of §3.(1&2): Propositions, forms, etc
- Demo SML features from §3.3: Boolean values
- Work through §3.4: Logical equivalences (Game 1)

Which phrase gives the best metaphor for the meaning of "set of sets"?

Champion of champions Horror of horrors

Box of boxes

Friend of a friend

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What is the cardinality of $\mathscr{P}(\emptyset)$?

If set X has cardinality n, then what is the cardinality of $\mathscr{P}(X)$?

A **proposition** is a sentence that is true or false, but not both.

It is snowing and it is not Thursday.

A propositional form is like a proposition but with content replaced by variables.

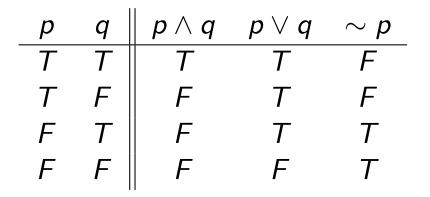
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p and not q

 $p\wedge \sim q$

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Evaluate (to T or F) this logical expression:

 $(T \land (\sim F \lor F)) \land (T \land T)$

Evaluate (to T or F) this logical expression:

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 $(T \lor F) \land \sim (F \land T)$

Evaluate (to T or F) this logical expression:

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(F \lor F \lor T) \land (\sim T \land F)
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Commutative laws:	$p \wedge q$	≡	$q \wedge p$	$p \lor q$	≡	$q \lor p$
Associative laws:	$(p \land q) \land r$	≡	$p \wedge (q \wedge r)$	$(p \lor q) \lor r$	≡	$p \lor (q \lor r)$
Distributive laws:	$p \wedge (q \vee r)$	=	$(p \land q) \lor (p \land r)$	$p \lor (q \land r)$	≡	$(p \lor q) \land (p \lor r)$
Absorption laws:	$p \wedge (p \vee q)$	=	p	$p \lor (p \land q)$	≡	p
Idempotent laws:	$p \wedge p$	≡	p	$p \lor p$	≡	p
Double negative law:	$\sim \sim p$	=	p			
DeMorgan's laws:	$\sim (p \wedge q)$	=	$\sim p ee \sim q$	$\sim (p \lor q)$	≡	$\sim p \wedge \sim q$
Negation laws:	$p \lor \sim p$	=	Т	$p\wedge\sim p$	=	F
Universal bound laws:	$p \lor T$	=	Т	$p \wedge F$	≡	F
Identity laws:	$p \wedge T$	=	p	$p \lor F$	=	p
Tautology and contradiction laws:	$\sim T$	≡	F	\sim F	=	Т

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Remember from high school algebra that there are "simplify" problems and "solve" problems.

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■ Simplify $3x(2+3x)^2 + 1$. $3x(2+3x)^2 + 1$ $= 3x(4+12x+9x^2) + 1$ $= 12x + 36x^2 + 27x^3 + 1$ $= 27x^3 + 36x^2 + 12x + 1$ ■ Solve 12x = 57 - 7x for x. 12x = 57 - 7x 19x = 57x = 3 Suppose we were to show that $\sim (\sim p \land q) \lor (p \lor \sim p) \equiv p \lor \sim q$.

Do this:

$$\begin{array}{l} \sim (\sim p \land q) \lor (p \land \sim p) \\ \equiv & \sim (\sim p \land q) \lor F \\ \equiv & \sim (\sim p \land q) \lor F \\ \equiv & \sim (\sim p \land q) \\ \equiv & p \lor \sim q \end{array}$$
 by negation law by identity law

Don't do this:

$$\begin{array}{rcl} \sim (\sim p \land q) \lor (p \land \sim p) &\equiv p \lor \sim q \\ \sim (\sim p \land q) \lor F &\equiv p \lor \sim q & \text{by negation law} \\ \sim (\sim p \land q) &\equiv p \lor \sim q & \text{by identity law} \\ p \lor \sim q &\equiv p \lor \sim q & \text{by De Morgan's} \end{array}$$

Semester roadmap:

Ch 1 & 2: Raw materials Ch 3: Formal logic —Test 1, Feb 12 — Ch 4: Proofs Ch 5: Relations — Test 2, Mar 20 — Ch 6: Self reference Ch 7: Functions — Test 3, Apr 19 — Chapter 3 roadmap:

Today: Logical equivalences (Game 1) Wednesday: Conditionals (SML) Friday: Arguments (Game 2) Next week Monday: Predicates and quantification (SML) Next week Wednesday: Quantified arguments (Game 3) Next week Friday: Review for test

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For next time:

Pg 102: 3.3.(5 & 6) Pg 105: 3.4.(2, 4, 8-12) (See Canvas for a note about 3.4.(2 & 4))

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Read 3.(5-7) Take quiz